Long-range dependence and multifractality in the term structure of LIBOR interest rates

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Abstract

In this paper we present evidence of long-range dependence in LIBOR interest rates. We study a data set from 2000 to 2005, for six different currencies and various maturities. Empirical results suggest that the degree of long-range dependence decreases with maturity, with the exception of interest rates on Japanese Yen and on Indonesian Rupiah. Furthermore, interest rates have a multifractal nature and the degree of multifractality is much stronger for Indonesia (emerging market). These findings suggest that interest rates derivatives should take these features into account. Furthermore, fixed income risk and portfolio management should incorporate long-range dependence in the modeling of interest rates.

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1. Introduction

The literature on long-range dependence applied to stock returns is abundant. Many researchers have reported long-range dependence in a variety of stock returns indices and individual stock returns\textsuperscript{1}. However, only a few studies have focused on interest rates and their dynamics.

Studies on interest rates are very important as they are prices that are used in the economy to determine the level of investment and savings. Furthermore, if interest rates are long-range dependent then traditional option pricing models for interest rate derivatives and other derivatives have to be revaluated to incorporate this type of dynamics.

So far there is not an universally accepted theory for the dynamics of interest rates. This paper contributes to the literature by studying the dynamics of interest rates and testing for long-range dependence in the term structure of London Interbank offered rates (LIBOR) interest rates, which are a worldwide used benchmark for interest rates\textsuperscript{2}. Our results suggest that interest rates have a strong degree of long-range dependence, which

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\textsuperscript{1}See Cajueiro and Tabak [1,2] for a discussion on long-range dependence on equity returns and volatility, respectively.

\textsuperscript{2}See for example Bouchad et al. [3].

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suggests that pricing models should incorporate long-range dependence. These results are important for fixed income portfolio and risk managers and suggest that they should incorporate long-range dependence in the modeling of interest rates. Furthermore, interest rate derivatives should take these features into account.

An additional feature of the paper is that it shows that LIBOR interest rates are multifractal, and that the degree of multifractality is much higher for the Indonesian case (emerging market).

The paper is organized as follows: Section 2 provides a brief literature review. Section 3 describes the methodology associated with testing for time-varying long-range dependence. Section 4 describes the data. Section 5 presents the results of applying the methodology. Finally, Section 6 offers a brief conclusion.

2. Brief literature review

To the best of our knowledge the first author to consider long memory in interest rates has been Backus and Zin [4], who find evidence of long memory in the 3-months zero coupon interest rates for the US. Besides finding evidence of long memory in nominal interest rates the authors show that allowing for long memory in the short-term interest rate improves the goodness of fit of yield curves. Therefore, they show that taking into account long-range dependence can improve forecasting models of interest rates.

McCarthy et al. [5] also find long-range dependence in US interest rates using wavelets. Di Matteo et al. [6] study LIBOR interest rates and find that interest rates series are non-Gaussian and follow a leptokurtic distribution. Therefore, usual option pricing models are not valid anymore, specially for short-term horizon derivatives. The authors suggest some deviations from the usual Black and Scholes [7] model to price derivative on LIBOR interest rates.

Barkoulas and Baum [8] have shown that Euroyen deposit rates and the Euroyen term premium possess long-range dependence for Japan. Tabak and Cajuieiro [9] have found that Japanese interest rates are also long-term dependent. The authors find that short-term interest rates are anti-persistent and suggest that this feature is due to the zero-interest rate policy that has been pursued by the Bank of Japan in order to boost the economy.

Di Matteo and Aste [10] have shown that Eurodollar interest rates show probability distributions with fat tails and non-Gaussian behaviors. They also show that collective motion of the interest rates curve can be analyzed in sub-groups of maturity dates with similar behaviors. Therefore, empirical findings suggest that studying the spectrum of interest rates for different maturities is important, due to the variety of interest rate behavior in different sub-group of maturities.

Overall, a small number of papers that deal with long memory on interest rates have found evidence of long-range dependence, which suggests that it is worthwhile investigating such properties for LIBOR interest rates. Furthermore, evaluating such results for different maturities is important to obtain insights on the term structure of interest rates.

3. Long-range dependence

There are several methods that may be used to take into account the long-range dependence phenomena. However, in spite of the existence of several methods, the task of calculation the Hurst exponent is not straightforward and the methods sometimes present incompatible estimations of the long-range dependence parameter.

In this paper, we follow a method introduced by Barabási and Vicsek [17] and used recently by Di Matteo et al. [18] to measure the degree of market development of several financial markets. According to Di Matteo et al. [18], it combines sensitivity to any type of dependence in the data and simplicity. Moreover, since it does not deal with max and min functions, it is less sensitive to outliers than the popular R/S statistics.

Let \( X(t) \) be the price of a stock on a time \( t \). The generalized Hurst exponent is a generalization of the approach proposed by Hurst. Barabási and Vicsek [17] suggest analyzing the \( q \)-order moments of the distribution of increments, which seems to be a good characterization of the statistical evolution of a
stochastic variable $X(t)$,

$$K_q(\tau) = \frac{\langle |X(t+\tau) - X(t)|^q \rangle}{\langle |X(t)|^q \rangle},$$

where the time-interval $\tau$ can vary. The generalized Hurst exponent can be defined from the scaling behavior of $K_q(\tau)$, which can be assumed to follow the relation

$$K_q(\tau) \sim \tau^{qH(q)},$$

In order to check for robustness of results we also employ the commonly used detrended fluctuation analysis (DFA). Besides, we employ the multi-fractal-detrended fluctuation analysis (MF-DFA) developed by Castro e Silva and Moreira [19] and investigated recently by Kantelhardt et al. [20] to test for multifractality for the interest rates object of this study. This method is an extension of the DFA developed independently by Moreira et al. [21] and Peng et al. [22] and used to determine the Hurst exponent of self-affine monofractals.

In order to introduce the MF-DFA, let $Y(t)$ be the integrated time series of logarithm returns, i.e., $Y(t) = \log(X(t))$, where $X(t)$ is the price of the asset. We consider the $\tau$-neighborhood around each point $Y(t)$ of the time series. The local trend in each $\tau$-size box is approximated by a polynomial of order $m$, namely $Z(t)$.

Then, one may evaluate the $q$-order fluctuation function, namely

$$w_q(Y, \tau) = \left\{ \frac{1}{\tau} \sum_{t \in \tau} [(Y(t) - Z(t))^2]^{q/2} \right\}^{1/q},$$

where $q$ can take any real value different from zero. It is easy to show [19,20] that

$$\langle w_q(Y, \tau) \rangle \sim \tau^{H(q)}.$$
5. Empirical results

It is well known that the statistical properties of financial time series are varying with time and they depend on time windows. We have performed unit root tests to test for non-stationarity of interest rates. Unit roots tests will indicate whether we can apply Eq. (1) to interest rates or whether we should work with changes in log interest rates, instead.

In Table 1 we present unit root tests for both levels and first differences in log interest rates. For all currencies and maturities we cannot reject the null of a unit root in the level of interest rates but we strongly reject it for first differences, which suggests that changes in log interest rates are stationary. Therefore, we proceed by testing the statistical properties of log changes in interest rates, which are stationary.7

Table 2 presents descriptive statistics for log changes in interest rates. The Jarque–Bera statistic tests the Gaussianity assumption. For all interest rates with different maturities and currencies the Gaussian distribution assumption is rejected. The average skewness is \(-0.62\), while average kurtosis is 22.26, which suggests that rejection of normality is due to fat tails in the data. We also present standard errors for skewness and kurtosis measures. These findings are in line with previous work of Di Matteo and Aste [18]. Interest rates for Indonesia show a similar pattern as those from developed economies.

Table 3 presents the results for testing for long-range dependence in LIBOR interest rates. The evidence for LIBOR interest rates is not consistent with the weak form of capital market efficiency, which implies unpredictability of future returns based on historical returns.

Empirical results obtained suggest high Hurst exponents for interest rates. An important feature of these results regards that Hurst exponents decrease monotonically according to maturity for all except the Japanese Yen (increase monotonically) and Indonesia (emerging market).

Figs. 1–3 presents Hurst exponents for a range of maturities (3–12 months). The predictability of short-term interest rates is expected if Central Banks follow inflation targets and smooth the target interest rate path to achieve these targets, which seems to be in conformity with current procedures of main Central Banks. This is the case of Australia, England, the US and the EU. Therefore, the downward trend in Hurst exponents is in line with this expectations. Longer-term interest rates are more unpredictable than short-term interest rates (which are set up by Central Banks).

In Fig. 4 we have plotted the curves \(k_q(t)\) versus \(t\) for the US libor rates and the OLS linear regressions for these curves. We have calculated the generalized Hurst exponents from the slopes of these straight lines.

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7This approach is similar to detrending methods, which are often used to mitigate non-stationary effects.
The case of interest rates on Japanese Yen differs from the rest. Tabak and Cajueiro [9] find similar results studying Japanese interest rates from 6 to 240 months (20 years). The authors suggest that these findings may be interpreted as a result of the near zero interest rate policy pursued by the Bank of Japan. Basically, the response of longer-term interest rates to changes in the short-term rate is small in the neighborhood of the zero lower bound (interest rates are constrained to be non-negative).

The case of interest rates for Indonesia is similar to Japan as it has an upward slope. However, the degree of long-range dependence is as high as that observed for developed economies. These results suggest that long-range dependence in interest rates is universal and many different interest rates series for both developed and emerging economies show this pattern.

Table 4 presents results for the MF-DFA methodology for $q = 1, \ldots, 6$. Qualitative results remain the same, with an upward slope curve for developed economies, with the exception of Japan and Indonesia (emerging market). Furthermore, these slopes are independent of $q$. The differences in Hurst exponents seem to be higher.
for the US case (range from 0.788 to 0.565 for $q = 1$). These results suggest that these differences are not spurious or due to error measures. An important additional comment is that the degree of multifractality of Indonesia is much higher that the ones found for developed economies.8

It is interesting to note that for all countries, the degree of multifractality is decreasing monotonically with maturity. A possible interpretation for this result is that short-term LIBOR interest rates are more sensible to changes in the stance of monetary policy and business cycle conditions than longer-term interest rates.

8It would be worthwhile investigating if we can assess the degree of market development using multifractal measures.
Figs. 5–10 present Hurst exponents according to maturity for $q = 1$ and $q = 2$. These figures suggest that the slope is independent of the $q$ chosen.

The empirical findings suggest that it is important to study interest rate dynamics and to take into account the full spectrum of maturities, due to the differences in the dynamics of such time series. These differences
may be due to a variety of points and further research should focus on explaining such differences. For example, certain parts of the yield curve may be more liquid than others and therefore, a liquidity premium could play a significant role for some maturities. Besides, if the uncertainty in the economy is high a term

![Graph](https://via.placeholder.com/150)

Fig. 5. DFA Hurst exponents for LIBOR for different maturities (q = 1).

The last column presents the degree of multifractality measured as ln(maximum(H(q)))/minimum(H(q))).
premium may also be significant. More studies on the microstructure of the bond markets would enhance substantially our understanding of these phenomena.

6. Conclusions

This paper presents empirical evidence of strong long-range in LIBOR interest rates for a variety of maturities and currencies. The term structure of Hurst exponents decreases with maturity, which is consistent
with the notion that longer-term interest rates are more unpredictable and random. However, this does not hold for the Japanese Yen in line with previous findings (See [9]) and with an emerging market (Indonesia).

We employ two different methodologies to estimate Hurst exponents, the Generalized Hurst exponent and the DFA. Both methods provide the same results, which reinforces our findings. Furthermore, with the use of the MF-DFA method we provide evidence of multifractality in these interest rates. Indonesia has the highest degree of multifractality, suggesting that emerging markets have a stronger degree of multifractality. An additional finding is that the degree of multifractality is monotonically decreasing with maturity for all countries.

Fig. 8. DFA Hurst exponents for LIBOR for different maturities ($q = 2$).

Fig. 9. DFA Hurst exponents for LIBOR for different maturities—Japanese yen ($q = 2$).
The development of consistent option pricing theory taking into account long-range dependence in mean interest rates seems to be an important route for future research. Furthermore, further research should focus on the microstructure of the bond markets. Liquidity and term premia may play a significant role in driving the results obtained in this paper. Therefore, controlling for these effects would deeper our understanding of the phenomena presented here. Future research could also study whether the degree of multifractality may be used to assess the market stage of development.

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