Long memory testing for Fed Funds Futures’ contracts

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Abstract

In this paper, the evaluation of the long memory in returns and volatilities of returns of the daily prices of closing of 6 future generic contracts of Fed Funds negotiated in the Chicago Board of Trade (CBOT) is performed. This evaluation is also made for the spreads between prices of these generic contracts for the evaluation of the transmission of the effects of shock in the interest rates for the various horizons of expectations until 6 months. The study uses the classical R/S analysis for the determination of the Hurst exponent and the bootstrap through moving blocks for the determination of the standard error of the exponent. Long memory for the returns and for the volatility of the returns of the studied future contracts was identified, which suggests that the Monetary Authority of the United States has been able to maintain the stability of the interest rates in spite of shocks, or that the American economy is not significantly affected by shocks. The results of this paper also suggest that the adjustments of the expected interest rates of the Fed Funds occur quickly for the various horizons, not presenting long memory for the returns of the spreads, but for volatility.

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1. Introduction

The identification of the persistence in the basic interest rates of an economy is a relevant issue for its agents, affecting areas from the development of the monetary policy of the country to the making of investment decisions. The monetary policy, usually, is implemented by the monetary authority through the establishment of goals for the short-term interest rates. These rates influence, through the expectations of the market agents, the interest rates for longer terms, forming the structure of term of these rates, which is used by the market in the making of investment decisions.

The behavior of a certain interest rate can be analyzed through the modeling of its generating process. In this case, formative factors of this rate are identified and the behavior of these factors is modeled individually and in groups. In the calibration of these models, measurements of the factors and of the effects that they cause are considered. These effects are regularly imprecise, such as the effects of the analysis of scenario in the formation of expectations. Another way of realizing this study is to directly study the movement of the interest rates, without the modeling of the factors.

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that cause it, in a “black box” approach. In that case, tests for the identification of the stylized types of processes that could cause the observed movement in the interest rates are made. In both approaches, uncertainties come up, which are caused by the method and by the studied phenomenon.

In the particular case of this study, the second approach will be adopted in the verification of the occurrence of persistence, (or long memory), in returns and volatilities of future contracts of interest rates of the Fed Funds. In the literature, long memory has been studied, by various methodologies, for interest rates in the spot market\(^1\) and in the future market.\(^2\) Gil-Alana [11], for example, identified long memory for monthly data of Fed Funds interest rates. In this paper, the persistence (or long memory) will be quantified in returns and volatilities of the first 6 future generic contracts of Fed Funds interest rates, negotiated at CBOT (Chicago Board of Trade), using the classic R/S analysis. For the determination of the standard error of the obtained measure, the post-blackening bootstrap of the used series will be performed. Additionally, the spread between these generics will be analyzed, also from the point of view of the returns and of their volatilities, with the goal of evaluating the behavior of the expectations of short-term interest rates during the studied period.

This paper is structured in the following manner. In the next section, the methodology used for estimation of long memory is introduced. In Section 3, the data and the context from which they were extracted is described. In Section 4, the results are presented. Finally, in Section 5, this paper is concluded.

2. Methodology

The evaluation of the long memory of a series can be made through various methodologies. Among the methodologies which are used today for the identification and the quantification of long memory, the ones most commonly used are the classic R/S analysis, developed by Hurst [16] and Mandelbrot [21], the modified R/S analysis due to Lo [18], the parameter of fractional integration estimation through a spectral regression or log-period due to Gewek and Porter-Hudak [10], the semi-parametical estimator through log-period due to Robinson [23] and the V/S analysis due to Giraitis et al. [12]. These methodologies can be used without prior knowledge of the factors that act in the price generating process, taking into consideration only the series of returns or volatilities (or residuals of these variables in relation to regressors) for which it is desired to estimate long memory. In general, these procedures result in parameters of long memory, whether it is the Hurst exponent “\(H\)”, or the parameter of fractional integration “\(d\)”, which can be used in the econometric modeling of processes with long memory at average, such as the ARFIMA\((k,d,l)\) (Auto Regressive Fractionally Integrated Moving Average), proposed by Granger and Joyeux [13] and Hosking [15].

In this paper, the long memory is measured by the Hurst exponent “\(H\)”, calculated through the classic R/S analysis. When \(0 < H < 0.5\), the analyzed series is anti-persistent, presenting reversion to the mean; if \(H = 0.5\), the series presents random walk and if \(0.5 < H < 1\), the series is persistent, with the maintenance of tendency. This methodology was initially proposed by Hurst [16] and then by Mandelbrot and Wallis [20,22] and Mandelbrot [19] verified that the empirical relation discovered by Hurst exhibited the same form as the one presented by the series that describe the Brownian fractional movement, regarding the rescaled range (R/S) in function to the period used in the calculus (\(\tau\)) and, therefore, that the Hurst exponent “\(H\)” could be used to represent long memory properties. The calculus of the Hurst exponent is done as follows: \(X(t)\) is the closing daily price of the day \(t\) of the future contract of Fed Funds to be studied and \(r(t)\) the logarithmic return of this rate at the given date, given by \(r_t = \ln(X(t)/X(t-1))\). For the long memory estimation for the volatilities series, the absolute value of \(r_t\)\(^4\) is used as an approximation for the instantaneous volatility in \(t\). In order to calculate the Hurst

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\(^1\) Backus and Zin [1] found evidences of long memory in the interest rates of zero coupon titles in the USA and verified that by taking into consideration this long memory, there is improvement in the adjustment of the mean and of the volatilities in the yield curves; Tsay [26] identified long memory in series of real interest rates ex post of the USA; Gil-Alana [11] studied annual data of long-term interest rates of the USA, identifying long memory for a sample of the years 1978–2000, and unitary root for the years 1940–2000.

\(^2\) Booth and Yiuman [2] examined the relationship between the domestic interest rate of the USA and the external interest rate, and did not identify long memory for the individual series, but verified that the errors of equilibrium possess long memory, and Thomakos and Wang [27] studied interdaily data of future contracts of Eurodollar and of American Treasury Bonds, not finding long memory in the returns, but finding long memory in the volatilities.

\(^3\) Actually, the value of the interest rate to be studied is obtained from: \(X(t) = 100 - P(t)\), being that \(P(t)\) = closing price of future contract of the Fed Funds at date \(t\).

\(^4\) Cajueiro and Tabak [3] when analyzing the predictability due to long memory of volatilities compared the usage, as an approximation of volatility, of the return to the square and of the absolute value of the return and concluded that, even though the approximation done through the absolute value gives higher values of the Hurst exponent through time, the results obtained by both are qualitatively equal.
exponent “H”, the calculations of the R/S statistic are made for diverse lengths of the τ block, given that τ ≤ N (number of observations in the series). At the end of these calculations, there is a table of τ values and R/S, corresponding statistics. The Hurst exponent “H” is obtained by the following expression, defined by Mandelbrot and Wallis [20]:

$$\log_{10}(R/S) = \log_{10}C + H\log_{10}τ + error$$

The obtainment of the standard error of “H” through the method by the least ordinary squares above presents difficulties, given that the data in different points (τ, R/S) are correlated, because they are constructed from aggregations of the same series of data. We try to estimate the standard error of “H” through the use of a bootstrap of the analyzed series. After that, the Wald test with the null hypothesis $H = 0.5$ in the identification of the long memory occurrence is used. The bootstrap is used to simulate the distribution of probabilities of any given statistic. This is done through the re-sampling of the original data with the objective of creating new data series, from which is obtained the statistic of interest. At the end of various simulations, the empirical distribution of the statistic of interest is obtained. In the case of data with dependency, the process of re-sampling should preserve the statistical structure of the original series, as stated by Grau-Carles [14] and Davison and Hinkley [6].

An interesting way of preserving the dependency structure of the studied series is through the post-blackening approach. In this approach, the original series is filtered by a model capable of capturing the dependence exhibited by the series, in a process known as pre-whitening. The residuals of this filter are centered and re-shown, and then, are added to the series generated by the estimated parameters for the model used in the filtering process, in the post-blackening process, resulting in a new series of data to used. Usually, an AR($p$) model is used, with p big enough to represent the dependency exhibited in the series. In this paper, the Akaike information criteria were used in the determination of the $p$ value, up to a maximum of 30. It was verified, through simulations, that the generating of 100 replicas of the original series by bootstrap is adequate to obtain the standard error of the Hurst exponent. In this paper, 1000 replicas were used, without any relevant differences in the obtained results.

The procedure considered here is given by the following steps:

1. The Hurst exponent for the original series is estimated, obtaining “H”.
2. The pre-whitening of the series is made, estimating an AR($p$) model with $p$ sufficiently high. The order of the autoregressive model is estimated through the Akaike information criteria. The residuals of the $e_i$ model and the centered residuals $\hat{e}_i - \bar{e}_i$ are obtained.
3. A new series of innovations through the bootstrap of moving blocks (MBB) is generated. In this process, a block length of the block = 5 observations is used, according to the Hall et al. [25] rule.

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5 The calculation of the R/S statistic for a given length of sample τ is made by the following manner:
Initially, the series of N returns is divided in n contiguous blocks of τ elements, numerated with $1 \leq i \leq n$. In each block $i$, the elements $r_i, \overline{r_i}$ are numerated with $1 \leq i \leq τ$.

The mean for the return of each block is calculated by $\overline{r_i} = \frac{1}{n} \sum_{j=1}^{n} r_{ij}$.
The standard deviation in each block is calculated by $S_i = \frac{1}{n} \sum_{j=1}^{n} (r_{ij} - \overline{r_i})^2$.

For each block $i$, the (R/S) statistic is calculated: $(R/S)_i = \frac{1}{2} \frac{1}{n} \max_{0 < \tau < N} \sum_{j=1}^{n} (r_{ij} - \overline{r_i}) - \min_{0 < \tau < N} \sum_{j=1}^{n} (r_{ij} - \overline{r_i})$.
The mean of the (R/S) value is calculated and is associated to the size of the block τ, obtaining values of $(τ, R/S)_i$: $(R/S)_i = \frac{1}{n} \sum_{i=1}^{n} (R/S)_i$.

6 The Post-Blackening was discussed in Freedman [9] and in Efron and Tibshirani [7,8], among other works.

7 In this method, it is assumed that the original series behaves according with the determined model, being obtained, through the estimation of its parameters, its residuals, that must behave as white noise. In case a representative model of the original series has not been chosen, the obtained residuals through this model will not be white noise and the bootstrap results will be compromised.

8 This technique was studied by Kunsch [17] and consists of the division of the original series in blocks of the same length, which are picked out with reposition and juxtaposed, in order to form a new series of the blocks that were picked out, with the same length as the original series. The blocks that participated of the random selection can be partially over-posed. Kunsch also studied the bootstrap of blocks of blocks, which consists of selecting randomly blocks formed in order to preserve the statistical structure of the original series. Srinivas and Srinivasan [24], when analyzing the use of MBB, made the following comments:

- The intra-block dependence structure is maintained, but, in the limits between blocks, it is destroyed, which compromises the results and requires the use of blocks of greater length. This limitation is overcome even when greater lengths of blocks are used, with over-posing.
- The simulations with the MBB do not allow the inter-poling between points of data in the historical register.
- The simulations do not allow the generation of values that are further from the mean than the ones that can be obtained from the observed series.
- After a determined number of simulations, there is no gain in quality.
4. The post-blackening is made, adding the innovations series generated by bootstrap to the model whose parameters were generated in the pre-whitening, to obtain the synthetic series.
5. For each synthetic series, the Hurst exponent $H_b$ is estimated.
6. At the end of the process, the Wald statistic is calculated given by $W = \frac{(H - 0.5)^2}{S(H)^2}$, $S(H)$ being the standard error of the Hurst exponents obtained through the bootstrap.

3. Data

This paper starts from the closing prices of the first 6 generic future contracts of Fed Funds interest rates of 30 days, negotiated at the Chicago Board of Trade (CBOT), obtained from Bloomberg.

The series of prices of the $n$th future generic contract are formed by the closing prices of the $n$th next due date of the analyzed future contract. This way, in the date following the due date of each contract of the series, we start using the closing prices of the due date after the one being used until then. These due dates are called “rolling dates” of the series of generic contracts. In this paper, the series of return and of volatilities of the returns of the generic contracts were filtrated with relation to the passing of the rolling dates to avoid possible distortions in the calculations of the Hurst exponent caused by the substitution of the series that occurs in the dates. The series of analyzed generic contracts are of daily data, in the period from December 6 1988 to May 31 2006 (4399 observations) and are referred to 6 generic contracts, denominated from now on, FF1 to FF6, the numbers from 1 to 6 indicators of the $n$th next due date that composes the series. The spread series of closing prices were also analyzed, between FF2 and FF1, FF3 and FF1, FF4 and FF1, FF5 and FF1 and FF6 and FF1. For the studied series, were calculated:

- Returns from: $r_t = \ln\left(\frac{X(t)}{X(t-1)}\right)$, $X_t$ and $X_{t-1}$ being the closing prices at dates $t$ and $t-1$;
- Volatilities $\nu_t = $ absolute value of $r_t$

4. Results

The Hurst exponents were calculated for future generic contracts 1–6 of Fed Funds, according to the methodology presented. The Hurst exponent $H$ was calculated through the classic R/S analysis, while the standard error and the confidence interval of 95% were calculated through moving blocks bootstrap (MBB). The results of these calculations, for returns and volatilities of returns, are presented in Table 1. For each contract, the Hurst exponent is presented, the standard error and the confidence interval of 95%, calculated by bootstrap and the $p$-value for the null hypothesis $H = 0.5$, obtained from the corresponding Wald statistic.

Long memory was identified in the returns and in their volatilities, as is confirmed in Fig. 1. The identified long memory is more accentuated for the volatilities than for the returns, always outside of the confidence intervals of the values found for the returns. The closing prices of the future generic contracts FF$n$ indicate the expected interest rates of the Fed Funds for 30 days at the end of $n$ months. It was verified that there are no significant

<table>
<thead>
<tr>
<th>Serie</th>
<th>Interest rate</th>
<th>Interest rate volatility</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
<td>Bootstrap std. error</td>
</tr>
<tr>
<td>FF1</td>
<td>0.665</td>
<td>0.025</td>
</tr>
<tr>
<td>FF2</td>
<td>0.699</td>
<td>0.025</td>
</tr>
<tr>
<td>FF3</td>
<td>0.692</td>
<td>0.026</td>
</tr>
<tr>
<td>FF4</td>
<td>0.677</td>
<td>0.026</td>
</tr>
<tr>
<td>FF5</td>
<td>0.666</td>
<td>0.026</td>
</tr>
<tr>
<td>FF6</td>
<td>0.661</td>
<td>0.026</td>
</tr>
</tbody>
</table>

FF$n$ is the series of the $n$th generic Fed Funds futures CBOT contract, i.e., the series of daily closing prices of the $n$th next maturing Fed Funds futures’ CBOT contract on each date. We employ the post-blackening bootstrap to estimate standard errors for Hurst exponents.
differences in the measures of long memory identified for the returns of the different generic contracts; for the volatilities, the measures of long memory grow with the number of months of expectation associated to the generic contract.

The long memory for the spreads between the expected interest rates was also measured and the values of the Hurst exponent and confidence interval were found and are presented in Table 2 and Fig. 2. With the exception of the spread between the expected interest rates for 1 and 2 months, which presents slight anti-persistence, it was verified that the spreads between the expectations among the other terms do not present long memory. The volatilities of these spreads present fairly accentuated long memory.

The analyzed volatilities present accentuated long memory, because of that, the usage of models, for future contracts of the Fed Funds and for their spreads, that take into consideration the long memory of the volatilities, is recommended.

<table>
<thead>
<tr>
<th>Serie</th>
<th>Spread return</th>
<th>Spread volatility</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
<td>Bootstrap std. error</td>
</tr>
<tr>
<td>FF1–FF2</td>
<td>0.442</td>
<td>0.024</td>
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<tr>
<td>FF1–FF3</td>
<td>0.498</td>
<td>0.024</td>
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<tr>
<td>FF1–FF4</td>
<td>0.522</td>
<td>0.025</td>
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<tr>
<td>FF1–FF5</td>
<td>0.525</td>
<td>0.024</td>
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<tr>
<td>F1–FF6</td>
<td>0.519</td>
<td>0.023</td>
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</table>

FF1–FFn is the series of FF1–FFn generic contracts spreads. We employ the post-blackening bootstrap to estimate standard errors for Hurst exponents.
5. Conclusions

In this paper, we have found empirical evidence of the presence of long range dependence in returns and volatilities of generic contracts of the CBOT and the absence of it in spreads between the expectations of future interest rates of future generic contracts of the Fed Funds interest rates.

The presence of long memory for the returns and volatilities suggests most shocks have little impact on the stability of the American economy, or that the monetary authority have been successful in the effort of maintaining the stability of the interest rates after the occurrence of shocks.

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