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Estimating the Fractional Order of Integration of Yields in the Brazilian Fixed Income Market

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This paper presents evidence that yields on the Brazilian fixed income market are fractionally integrated, and compares the period before and after the implementation of the Inflation Targeting (IT) regime. The paper employs the commonly used GPH estimator and recently developed wavelets-based estimator of long memory. Empirical results suggest that interest rates are fractionally integrated and that interest rate spreads are fractionally integrated, with a higher order of integration in the period after the implementation of the IT regime. These results have important implications for the development of macroeconomic models for the Brazilian economy and for long-term forecasting. Furthermore, they imply that shocks to interest rates are long-lived.

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1. Introduction

The analysis of the dynamics of interest rates and of the term structure of interest rates is crucial for macroeconomics and finance. Most financial decisions depend on the current level of interest rates (for different maturities) and also of the expected future path for these interest rates. Therefore, a large body of literature has focused on studying the dynamics of interest rates (Rose, 1988; Stock and Watson, 1988; Campbell and Shiller, 1991; Wu and Chen, 2001).

One of the main topics in this research agenda is whether interest rates are stationary or not. Most theoretical models assume that interest rates are mean reverting (Brennan and Schwartz, 1980; Chan *et al.*, 1992). However, empirical literature has found evidence contradicting this hypothesis and suggesting that interest rates follow a random walk process (see Tse, 1995;

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Aquila *et al.*, 2003 among others). Therefore, there is no consensus on whether interest rates are a stationary stochastic process or not.

Recent literature has suggested an integrated process of order d, with $d < 1$, may describe well the dynamics of interest rates (Tkacz, 2001; Gil-Alana, 2004a, 2004b). In this case, interest rates may be stationary although shocks may be long-lived. The classical methods for testing unit root against trend-stationary representations (e.g. Dickey and Fuller, 1979; Phillips and Perron, 1988 or Kwiatkowski *et al.*, 1992) are seriously biased and present very low power if the alternatives are of a fractional form (see, e.g. Diebold and Rudebusch, 1991 and Hassler and Wolters, 1994).[1]

A few papers have provided convincing evidence that interest rates for the US have long memory.[2] Backus and Zin (1993) propose to model the interest rate for the US as a fractionally integrated log-normal random process. They provide evidence that their model provides a good description of interest rate data.[3] Gil-Alana (2004a, 2004b) tests for the order of integration in US interest rates and finds that mean reversion occurs for the short-term rates but tend to disappear with the term to maturity.[4] McCarthy *et al.* (2004) also study the dynamic behaviour of US interest rates using a wavelets approach and find evidence of long-range dependence in these time series.

Although a few papers discuss the dynamics of interest rates for the US, very little is known regarding the behaviour of interest rates in emerging markets. Gil-Alana (2003a) studies the Asian fixed income market and finds that short-term interest rates in Singapore and Thailand are long-run mean reverting, while results for Malaysia, South Korea and Philippines oscillate around the unit-root case. This suggests that it is not clear whether interest rates for emerging market possess fractional dynamics. This is an important issue as it directly impacts how to model the macroeconomic environment and develop policy and forecasting models.[5]

The purpose of this paper is to investigate the dynamics of the Brazilian term structure of interest rates. We also compare the periods before and after

[1] See Baillie (1996) for a nice review of *I(d)* processes and Beran (1994), Doukhan *et al.* (2003) and Robinson (2003) for excellent reviews of long memory.

[2] See Lo (1991), Booth and Tse (1995), Pfann *et al.* (1996) and Gil-Alana and Robinson (1997).

[3] The authors speculate that the long-memory property may derive from a fractionally integrated dynamic for inflation and/or the money growth rate.

[4] Parke (1999) justifies the presence of long memory in terms of the duration of shocks. See also Tkacz (2001). Shea (1991) investigated the consequences of long memory in interest rates for tests of the expectation hypothesis of the term structure, finding that allowing for the possibility of long memory significantly improves the performance of the model, even though the expectation hypothesis cannot be fully resurrected. See also Lai (1997), Phillips (1998), Tsay (2000), Meade and Maier (2003) and Couchman *et al.* (2006).

[5] For example, it impacts the formulation of VAR and VEC models currently in use to test economic relationships and perform economic and financial forecasts in most emerging economies. If interest rates are fractionally integrated then such models have to incorporate this feature.

the implementation of the Inflation Targeting (IT) framework for monetary policy. We employ recently developed fractional differencing wavelets-based tests in the paper (Jensen, 1999; Tkacz, 2001). These results are also compared with more traditional tests such as the one proposed by the Geweke and Porter-Hudak (GPH, 1983).

Empirical results suggest that the order of integration differs within monetary policy frameworks. This implies that the design of monetary policy and its instruments determine, at least partially, the order of integration of interest rates. To the best of our knowledge, this is the first paper that discusses how the order of integration changes with the conduct of monetary policy, and it is a first step in enhancing our understanding of the causes of long memory.

The outline of the paper is as follows. Section 2 briefly describes the testing procedure. In Section 3, the data are presented, while Section 4 provides the empirical results for the Brazilian term structure of interest rates. Section 5 contains some concluding comments.

2. Methodology

An I(0) process may be defined as a covariance stationary process with a spectral density function that is positive and finite at the zero frequency. In this context, an I(d) model is a process of the form:

$$(1 - L)^d x_t = u_t,\ t = 1, 2 \ldots \quad (1)$$

where u_t is assumed to be I(0), for any real value d, and L is the lag operator. Therefore, u_t may be a white noise process, an ARMA process (in which case x_t is ARFIMA) or any other suitable I(0) specification.[6] If $d > 0$, x_t is said to be a long-memory process.

Geweke and Porter-Hudak (1983), GPH hereafter, proposed a frequency domain estimator for the long-memory parameter d. The GPH estimator is a semi-parametric test for fractional processes that does not require any specification of the short-memory process.

Let $I(w_j)$ denote the sample periodogram at the jth Fourier frequency, evaluated at $w_j = 2\pi j / T$, with $j = 1, 2, \ldots T/2$.[7] The GPH estimator of d is based on the OLS regression of the log periodogram on the log frequency

$$\ln\left[I\left(w_j\right)\right] = \beta_0 + \beta_1 \log\left(w_j\right) + \varepsilon_j \quad (2)$$

[6] See Granger (1980) for a pioneering work on fractional integration and Granger and Joyeux (1980) and Hosking (1981) for a discussion on ARFIMA models.

[7] The periodogram is given by $I(w) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T}(x_t - \bar{x})e^{iwt}\right|^2 = \frac{1}{2\pi T}\sum_{t=1}^{T}\sum_{s=1}^{T}(x_t - \bar{x})(x_s - \bar{x})e^{iw(t-s)}$, where x_t corresponds to the time series being evaluated.

where $j = 1, 2 \ldots J$ and $\hat{d} = -\frac{\hat{\beta}_1}{2}$. In our applications, we follow the literature and fix J equal to $T^{0.5}$ and $T^{0.6}$ to check for robustness.

The GPH estimator is known as a semi-parametric estimator because it yields an estimate of the fractional integration parameter, d, without specification of the short-term dynamics. This is a considerable advantage over parametric methods because misspecification of the short-term dynamics does not bias the estimate of the fractional integration parameter.[8] However, Jensen (1999) suggests an alternative, a wavelet-based estimator, that has a smaller mean squared error than the GPH estimator for both small and large sample sizes and different values of d.[9]

A function $f(x)$ can be expressed in the wavelet domain as:

$$f(x) = \alpha + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j - 1} w_{j,k} \Psi\left(2^j x - k\right) \tag{3}$$

where the index j is the scaling index, which compresses function $\Psi(x)$ and the index k is the translation index, which shifts function $\Psi(x)$, and $w_{j,k}$ are the wavelets coefficients. These coefficients ($w_{j,k}$) represent how much information is lost (or gained) if the series x_t is sampled less (or more) often. The wavelets coefficients are distributed as $N(0, \sigma^2\ 2^{-2jd})$.

The Haar wavelet is one of the simplest examples for the mother wavelet $\Psi(x)$ in expression (3), [10] which is given by:

$$H(x) = \begin{cases} 1 & 0 \leq x \leq 1/2 \\ -1 & 1/2 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

The main advantage of this wavelet is the finite support property and the simplicity of the coefficient computation procedure. Daubechies wavelets (1988) have more desirable properties, such as improved frequency localization and the ability to represent continuous signals more parsimoniously.[11]

Jensen's (1999) wavelet estimator employs the autocovariance function of the wavelet coefficients to estimate the fractional differencing parameter.[12] Define *R(j)* to be the wavelet coefficient's variance at scale j ($R(j) = \sigma^2\ 2^{-2jd}$). Taking the logarithmic transformation of *R(j)*, the following

[8] The GPH procedure was improved by Kunsch (1986) and Robinson (1995). Velasco (1999) extended the GPH to the case of non-stationary series by means of tapering. See, also, Shimotsu and Phillips (2002) and Kim and Phillips (2006) for further refinements of this approach.

[9] The strength of the wavelets approach relies in its ability to simultaneously localize a process in time and scale. Furthermore, the wavelet OLS estimator yields a consistent estimator of the fractional differencing parameter.

[10] See Daubechies (1992), Strang (1993), Ojanen (1998) and Gencay *et al.* (2002).

[11] However, they cannot be expressed in a closed form algebraic expression.

[12] If the process has long memory then the autocovariance function shows hyperbolic decay.

relationship is obtained:

$$\ln\left(R_j\right) = \ln\sigma^2 - d\ln 2^{2j} \tag{4}$$

where the fractional differencing parameter, d, can be estimated by OLS.

The value of the parameter d indicates whether a time series follows a long-memory process. If $d = 0$, the time series is said to be a short-memory and stationary process (no long memory is present) and if d takes non-integer values the process is said to be fractionally integrated, i.e. for any d strictly higher than 0 long memory occurs (including the unit root case ($d = 1$) and $d > 1$). For $0 < d < 0.5$, the series is said to be a long-memory process with finite variance (stationary), while for $0.5 \leq d < 1$ the series has an infinite variance and long memory. If $d > 1$ the series is defined as a strong long-memory processes. In this case, past shocks to the series have permanent effects but the first differenced series is stationary with a long-memory autocovariance function.

3. Data

The main data employed in this study are interest rates swaps maturing in 1, 3, 6 and 12 months' time and the Selic interest rate, which is the benchmark interest rate used for monetary policy purposes by the Central Bank of Brazil.

In the swap contracts, a party pays a fixed rate over an agreed principal and receives a floating rate over the same principal, the reverse occurring with his counterpart. There are no intermediate cash flows, with contracts being settled on maturity. We use the fixed rate, negotiated by the parties.[13]

The data are sampled daily, beginning on 10 April 1995 and ending on 17 July 2003, with 2048 observations in the full sample.[14] We also study the behaviour of interest rates for two sub-periods, coinciding with the implementation of the IT framework for monetary policy. The first sub-period, pre-IT begins on 10 April 1995 and ends on 31 May 1999, while the second sub-period, post-IT, begins on 1 June 1999 and ends on 17 July 2003. We have 1024 observations in each one of these sub-periods.

Figure 1 presents the dynamics of the time series under study. In March 1995, interest rates went up as a consequence of the Mexican crisis. Interest rates also went up in the Asian (1997) and Russian (1998) crises. In the period prior to 1999, the Brazilian Central Bank followed a crawling-peg

[13] The floating rate is the overnight CDI rate (interbank deposits), which tracks very closely the average rate in the market for overnight reserves at the Central Bank.

[14] The selection of the sample depends on data availability.

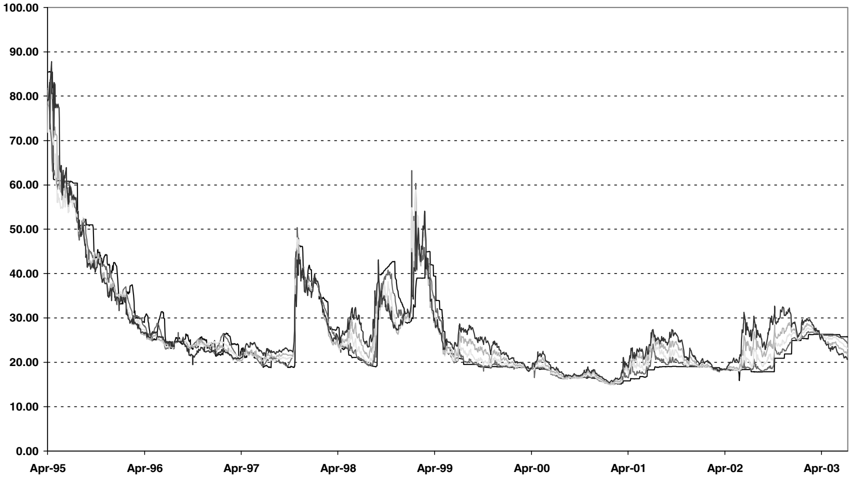

Figure 1: Interest Rates for the Brazilian Economy (Selic, 1, 3, 6 and 12-Months Interest Rates)

exchange regime, and in crisis periods interest rates went up to absorb external shocks.[15]

Table 1 presents descriptive statistics for the spectrum of Brazilian interest rates. It is worth noting that the period from 1995 to 1999 is characterized by high volatility in interest rates if compared to the period after the implementation of the IT (1999–2003).[16] In the former period, the exchange rate was fixed and followed a crawling peg regime, with a small devaluation occurring during the year. Therefore, external shocks had to be absorbed by interest rates. In the event of crises, yields on emerging markets sovereign bonds increase and give rise to arbitrage opportunities, which leads to outflows of capital in emerging countries. In this case, one has to increase domestic interest rates in order to reduce arbitrage movements in line with yields on sovereign bonds. After the adoption of a floating exchange rate regime, however, part of the adjustment in such events can be absorbed by exchange rate fluctuations, which reduces the volatility of domestic interest rates.

We also study the behaviour of interest rates spreads. Figure 2 presents the dynamics of the spread between the 12-months and the Selic interest

[15] In these turbulent periods, yields on external Brazilian debt went up. This happened mainly because there were concerns in the market as to whether the Central Bank would be able to defend the currency peg, while led to outflows of capital, with substantial reduction of international reserves.

[16] The Selic interest rate has the highest standard deviation in the pre-IT regime and the lowest standard deviation in the post-IT regime. Therefore, for the whole sample the Selic interest rates have the highest standard deviation.

Table 1: Descriptive Statistics

	SELIC	1-month	3-months	6-months	12-months
Panel A: Full Sample					
Mean	26.05	25.91	26.00	26.55	27.45
Median	21.20	22.07	22.93	23.74	24.69
Std. Dev.	11.55	10.71	10.24	10.30	10.51
Skewness	2.00	1.88	2.15	2.41	2.58
Kurtosis	7.69	6.58	8.45	10.06	11.56
JB	3243.8	2304.9	4113.0	6226.8	8529.8
p-value	0.00	0.00	0.00	0.00	0.00
Panel B: Pre-IT					
Mean	32.77	32.10	31.57	31.70	32.23
Median	28.96	27.94	26.33	26.46	27.65
Std. Dev.	12.93	11.99	11.74	12.15	12.58
Skewness	1.36	1.22	1.52	1.76	1.99
Kurtosis	5.10	4.04	5.34	6.19	7.23
JB	505.5	300.3	629.7	960.9	1438.3
p-value	0.00	0.00	0.00	0.00	0.00
Panel C: Post-IT					
Mean	19.33	19.73	20.44	21.40	22.66
Median	18.87	18.97	19.71	21.25	22.19
Std. Dev.	3.00	3.01	3.12	3.41	4.11
Skewness	1.18	0.95	0.54	0.17	0.25
Kurtosis	3.61	3.12	2.52	2.11	2.12
JB	254.9	154.6	60.1	39.0	44.4
p-value	0.00	0.00	0.00	0.00	0.00

JB stands for the Jarque-Bera normality test. In all cases, the *p*-value for the JB statistic suggests that the normality assumption should be rejected. We present statistics on the excess Kurtosis.

rates. This spread is positive on average (1.4 per cent), suggesting that most of the time the term structure has a positive slope.

4. Empirical Results

Table 2 presents the estimates of the fractional integration parameters for Brazilian interest rates for the full sample of observations and for the sub-periods, defined according to the monetary policy framework. Jensen (1999) and Tkacz (2001) suggest the use of Haar and Daubechies wavelets for the analysis of time series. We employ Haar and Daubechies wavelets (Daubechies, 1988) and test for robustness by using two smoothing parameters to ensure robustness (Daubechies 4 and 12). Interest rate series are also examined for a fractional exponent in the differencing process using the GPH test. The unit-root hypothesis is tested by determining whether the GPH estimate of d is significantly different from one.

For the full sample, the GPH estimates suggest the presence of fractional integration with $d < 1$ for the 6-months period and the presence of

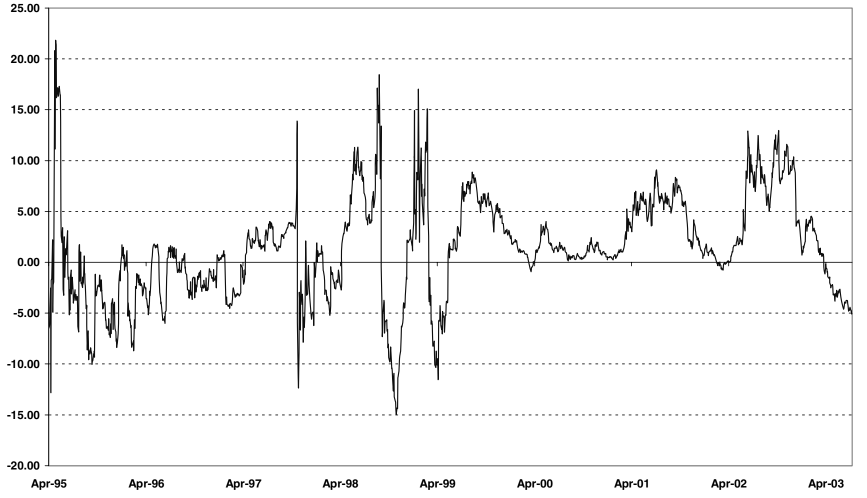

Figure 2: Interest Rates Spreads (12-Months – Selic) for the Brazilian Economy

a unit root for the remainder of interest rates. However, from the wavelets-based estimates we cannot reject the presence of long memory in interest rates for all maturities.[17] An interesting finding is that interest rates possess positive long-range dependence and are non-stationary processes as $d > 0.5$. Furthermore, persistence does not increase with interest rates' maturities. On the contrary it decreases with maturity, which suggests that the very short-term interest rates are the most persistent.

When we compare two sub-samples, the period prior to and after the implementation of the IT regime, results change. In the period prior to the IT regime, interest rates are fractionally integrated. However, in the period after the IT we cannot reject that the Selic interest rate is a unit-root process. For the long-maturity (12-months) interest rates results also suggest a unit-root process after the IT. Therefore, the degree of integration has changed for very short-term and long-term interest rates within these time periods.

Tkacz (2001) finds that the evidence of non-stationarity in long-term rates has diminished in the recent period (after 1991) and that nominal rates for the US follow fractionally integrated processes. Furthermore, for Canada the author finds that long-term interest rates are more non-stationary than short-term, which may be due to term premia.

Evidence for the Brazilian term structure of interest rates shows very different results. Short-term interest rates seem to be more non-stationary

[17] Jensen (1999) has shown that the mean squared error of the wavelets estimator of the long-memory parameter is four to six times smaller than the mean squared error of the GPH estimator.

Table 2: Fractional Integration Parameters (d) for Brazilian Interest Rates for Different Time Periods

	Haar	Daubechies-4	Daubechies-12	GPH $T^{0.5}$	GPH $T^{0.6}$
Full Sample: April 1995 to July 2003					
Selic	0.8765	0.7414	0.7781	0.84127*	0.78166
	(0.0521)	(0.0855)	(0.0798)	(0.0933)	(0.0605)
1-month	0.8465	0.7284	0.7582	0.92958*	0.89154*
	(0.0488)	(0.0801)	(0.0737)	(0.1111)	(0.0652)
3-months	0.8432	0.7153	0.7471	0.80444*	0.81216
	(0.0507)	(0.0975)	(0.0873)	(0.0930)	(0.0503)
6-months	0.845	0.6572	0.7078	0.76109	0.85961
	(0.0520)	(0.1247)	(0.1057)	(0.0833)	(0.0466)
12-months	0.8407	0.597	0.6711	0.72123	0.91601*
	(0.0585)	(0.1574)	(0.1252)	(0.0677)	(0.0439)
April 1995 to May 1999 – Pre-IT					
Selic	0.8763*	0.808	0.8257	0.7589*	0.72032
	(0.0666)	(0.0472)	(0.0543)	(0.1245)	(0.0669)
1-month	0.8385	0.7941	0.8083	0.89615*	0.86344
	(0.0667)	(0.0350)	(0.0386)	(0.1059)	(0.0653)
3-months	0.8432	0.8182	0.8277	0.75331	0.83834
	(0.0707)	(0.0456)	(0.0479)	(0.0837)	(0.0544)
6-months	0.8556	0.8221	0.8363	0.74891	0.91987*
	(0.0710)	(0.0450)	(0.0463)	(0.0747)	(0.0573)
12-months	0.8502*	0.834	0.8507	0.81486	0.9235*
	(0.0826)	(0.0451)	(0.0435)	(0.0836)	(0.0652)
June 1999 to July 2003 – Post-IT					
Selic	1.0509*	1.0501*	1.0649*	1.32507*	1.1306*
	(0.0356)	(0.0510)	(0.0493)	(0.0980)	(0.0830)
1-month	0.9043	0.9289	0.9362*	1.1876*	0.96831*
	(0.0423)	(0.0330)	(0.0331)	(0.1765)	(0.1141)
3-months	0.8641	0.8936	0.8996	1.0251*	0.83976*
	(0.0286)	(0.0267)	(0.0242)	(0.1886)	(0.1082)
6-months	0.8591	0.8789	0.8892	1.06339*	0.95691*
	(0.0203)	(0.0459)	(0.0372)	(0.1394)	(0.0882)
12-months	0.8761	0.8584*	0.8815*	1.00899*	1.03381*
	(0.0269)	(0.0855)	(0.0674)	(0.1470)	(0.1061)

*Unit root is within two standard errors of the estimated *d*.

than medium-term and long-term interest rates. An important issue is that in the case of the Brazilian economy interest rates maturities are considered only up to 12 months, due to the lack of longer maturity time series.

We also test for fractional integration of the spread between different interest rates. Results are presented in Table 3. In almost all cases, interest rates spreads are fractionally integrated with an integration order lower than those of the original series. These results suggest that interest rates spreads possess long memory. It is observed higher order of integration in the post-IT period, implying that the degree of non-stationarity or the effects of the shocks seem to be longer after the IT break.

Table 3: Fractional Integration Parameters (d) for Interest Rates Spreads for Different Time Periods

	Haar	Daubechies-4	Daubechies-12	GPH $T^{0.5}$	GPH $T^{0.6}$
Full Sample: April 1995 to July 2003					
1-month – Selic	0.4323	0.283	0.347	0.16999	0.4643
	(0.0452)	(0.1105)	(0.0888)	(0.0852)	(0.0714)
3-months – Selic	0.5039	0.4946	0.511	0.46842	0.63864
	(0.0463)	(0.0549)	(0.0552)	(0.1206)	(0.0712)
6-months – Selic	0.5116	0.5619	0.5791	0.51028	0.65373
	(0.0687)	(0.0471)	(0.0494)	(0.1217)	(0.0701)
12-months – Selic	0.5528	0.605	0.6279	0.49248	0.70216
	(0.0751)	(0.0530)	(0.0539)	(0.1822)	(0.0985)
3-months – 1-month	0.436	0.5143	0.5248	0.31307	0.51765
	(0.0782)	(0.0462)	(0.0510)	(0.0971)	(0.0600)
6-months – 1-month	0.4333	0.5706	0.5841	0.36657	0.58013
	(0.1136)	(0.0543)	(0.0556)	(0.1287)	(0.0750)
12-months – 1-month	0.5567	0.627	0.6475	0.40819	0.7814
	(0.0783)	(0.0591)	(0.0590)	(0.1152)	(0.0788)
6-months – 3-months	0.4193	0.5016	0.5225	0.4574	0.65634
	(0.1032)	(0.0554)	(0.0539)	(0.1193)	(0.0712)
12-months – 3-months	0.5468	0.5931	0.6154	0.45561	0.72359
	(0.0734)	(0.0623)	(0.0637)	(0.0891)	(0.0586)
12-months – 6-months	0.5598	0.5528	0.5654	0.42143	0.53299
	(0.0596)	(0.0689)	(0.0671)	(0.0923)	(0.0557)
April 1995 to May 1999 – Pre-IT					
1-month – Selic	0.4334	−0.1359#	0.2893	0.33895	0.5282
	(0.0579)	(0.3490)	(0.1345)	(0.1115)	(0.0752)
3-months – Selic	0.459	0.3293	0.0973#	0.38962	0.60398
	(0.0634)	(0.1372)	(0.2750)	(0.1339)	(0.0886)
6-months – Selic	0.4323	0.3533	0.4159	0.395	0.58403
	(0.0766)	(0.1292)	(0.1112)	(0.1387)	(0.0881)
12-months – Selic	0.4449	0.4961	0.5218	0.39	0.617
	(0.1068)	(0.0913)	(0.0955)	(0.1348)	(0.0816)

3-months – 1-month	0.3291	0.353	0.3491	0.13788#	0.54955
	(0.0866)	(0.1039)	(0.1155)	(0.1228)	(0.1006)
6-months – 1-month	0.2579	0.3708	0.4093	0.26849	0.61513
	(0.1443)	(0.1149)	(0.1073)	(0.1367)	(0.0926)
12-months – 1-month	0.4686	0.5171	0.5347	0.38474	0.75858
	(0.0977)	(0.0935)	(0.0963)	(0.1303)	(0.1063)
6-months – 3-months	0.3569	0.3906	0.4298	0.51166	0.72412
	(0.1113)	(0.0834)	(0.0810)	(0.1317)	(0.0844)
12-months – 3-months	0.5107	0.5057	0.5017	0.51018	0.82067
	(0.0832)	(0.1008)	(0.1210)	(0.1202)	(0.0884)
12-months – 6-months	0.546	0.5324	0.5061	0.25202	0.42732
	(0.0710)	(0.0862)	(0.1068)	(0.1100)	(0.0627)
June 1999 to July 2003 – Post-IT					
1-month – Selic	0.4784	0.5296	0.19	0.58205	0.46052
	(0.0765)	(0.0647)	(0.2621)	(0.1617)	(0.0971)
3-months – Selic	0.431	0.6066	0.58	0.67428	0.70399
	(0.1788)	(0.0931)	(0.1116)	(0.1375)	(0.0902)
6-months – Selic	0.5991	0.6245	0.6973	0.7562*	0.81609*
	(0.1118)	(0.1297)	(0.0953)	(0.1612)	(0.1037)
12-months – Selic	0.6947	0.6628	0.7927	0.76872*	0.84543*
	(0.1020)	(0.1473)	(0.0825)	(0.1708)	(0.0971)
3-months – 1-month	0.5452	0.5586	0.6319	0.66074	0.79528
	(0.1085)	(0.1230)	(0.0910)	(0.1333)	(0.0847)
6-months – 1-month	0.6116	0.5045	0.7351	0.7586*	0.94155*
	(0.1300)	(0.2100)	(0.0942)	(0.1418)	(0.1071)
12-months – 1-month	0.7112	0.7397	0.816	0.81667*	0.98824*
	(0.1101)	(0.1189)	(0.0826)	(0.1718)	(0.1179)
6-months – 3-months	0.4239	0.5997	0.7495	0.88662*	1.00257*
	(0.2384)	(0.1648)	(0.0920)	(0.1490)	(0.1038)
12-months – 3-months	0.7048	0.775	0.8298	0.85243*	0.99368*
	(0.1121)	(0.1009)	(0.0785)	(0.1444)	(0.1072)
12-months – 6-months	0.7224	0.7863	0.8231	0.80281*	0.91977*
	(0.0873)	(0.0762)	(0.0677)	(0.1165)	(0.0854)

*Unit root is within two standard errors of the estimated d. # is presented in cases in which we cannot reject the null of $d = 0$.

This test could be seen as a cointegration test with the imposition of a cointegrating vector (1, −1), which may be a restrictive assumption. With this caveat in mind, long-run relationship between short- and long-term interest rates may exist, which implies the error correction term in vector autoregression models may possess long memory, which means that deviations from equilibrium may be highly persistent.

5. Conclusions

Empirical results suggest that the Brazilian term structure of interest rates and individual yields are fractionally integrated, which suggests that shocks to interest rates are long-lived. Empirical results differ within monetary policy frameworks, which implies that the design of monetary policy and its instruments determine, at least partially, the order of integration of interest rates. It is worth mentioning that we have imposed an *a priori* break due to changes in monetary policy regime (implementation of an IT regime). Further research could exploit endogenously determined structural breaks (Gil-Alana, 2008).[18]

One possible explanation to the findings that interest rates are fractionally integrated is the periodic interventions of the monetary authorities, which affect the level or direction of interest rates. Furthermore, if fundamentals such as inflation, money growth among others possess long memory then we would expect interest rates to be fractionally integrated. Further research could focus on other variables (macroeconomic and financial) and study their dynamic behaviour.

Interest rates spreads are also fractionally integrated, which is a first indication that these interest rates may be fractionally cointegrated. Therefore, macroeconomic models should take these features into account for both modelling and forecasting purposes. Further research could exploit fractional cointegration tests such as the ones proposed in Robinson and Marinucci (2003), Gil-Alana (2003b), Chen and Hurvich (2003) and Robinson and Hualde (2003).

The findings obtained in the paper are important for the forecasting and modelling of interest rates. The construction of nonlinear models that takes these features in consideration may provide improvements in long-term forecasting exercise. The long-memory component should be taken into account when analysing interest rates for different maturities.

The study of other emerging markets is also worthwhile, as it would allow comparisons among countries and gaining a better understanding of the sources of long memory in financial variables.

[18] This is not a minor issue as recent literature has shown that neglecting occasional breaks may lead to spurious finding of long-memory properties (see, e.g. Beran and Terrin (1996), Kuan and Hsu (1998), Lobato and Savin (1998), Bos *et al.* (1999), Engle and Smith (1999), Diebold and Inoue (2001) and Granger and Hyung (2004)).

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Non-technical Summary

This paper presents evidence that yields on Brazilian fixed income market have long memory, and compares the period before and after the implementation of the IT regime, which has been in place in Brazil since mid-1999. With the IT, Brazil adopted a floating exchange rate regime and abandoned the crawling peg regime.

A time series is said to have long memory when the autocorrelation at long lags is not negligible and therefore, shocks that affect yields may last for a long time. The study of the degree of long memory is important as it has many implications for the development of macroeconomic and financial models.

The paper employs the commonly used Geweke-Porter-Hudak (GPH) estimator and recently developed wavelets' based estimator of long memory. A wavelets estimator is used as recent research has found that this estimator has better performance in the detection of long memory than traditional tests such as the GPH.

Empirical results suggest that interest rates have long memory and that interest rates spreads have long memory as well, with a higher degree of fractional integration in the period after the implementation of the IT regime. Therefore, the introduction of IT regime has provoked a structural break in the dynamics of interest rates.

These results have important implications for the development of macroeconomic models for the Brazilian economy and for long-term forecasting as they imply that shocks to interest rates are long-lived. Furthermore, they are important for the design of risk management models as interest rates forecasts in the long run have to incorporate long-memory considerations.

Further research could exploit whether these results hold for countries that have adopted IT regimes and compare developed and emerging market economies.