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Testing for time-varying long-range dependence in volatility for emerging markets

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Abstract

This paper tests whether volatility for equity returns for emerging markets possesses long-range dependence. Furthermore, the assertion of whether long-range dependence is time-varying is checked through a rolling sample approach. The empirical results suggest that there exists long-range dependence in emerging equity returns' volatility and also that it is time-varying. This assertion also holds true for Japan and the US, which are considered more developed markets. Moreover, these results are robust to "shuffling" the data to eliminate short-term autocorrelation. Therefore, they suggest that the class of GARCH processes, which are currently employed to analyze volatility of financial time series, is misspecified.

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Keywords: Emerging markets; Hurst exponent; Long-range dependence; Volatility

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1. Introduction

The presence of long-memory dependence in asset returns has been discussed extensively in the literature as it has particular implications for modeling the behavior of market asset prices. If asset returns do not follow a random walk behavior, then one can argue that investment risk is a function (among other factors) of the investor's preferred investment horizon. It also has significant implications on option pricing since the Black and Scholes option pricing model [1] may not be applied anymore.

There are many examples of empirical results for testing for long-range dependence in the literature. For example, in Ref. [2] empirical evidence of long-range dependence for developed economies is investigated and while no temporal dependence in returns is found, strong long-range correlations in conditional volatility is. Additionally, in Ref. [3] long-range dependence in the conditional variance for US equity return indices is tested and evidence of long memory is found. Furthermore, in Ref. [4] evidence of long-range dependence for the returns in commodity and foreign currency futures prices is presented.[2]

In a recent paper [8], it was shown that there seems to be long-range dependence in equity indices returns for a class of emerging markets indices. Furthermore, these long-range correlations were presented as time-varying and while for some countries there seemed to be a trend toward more efficient markets, the opposite held true for the other countries. In this paper, we address the same question of analyzing absolute returns to test this assertion for volatility.[3] Moreover, if volatility presents long-memory dependence and it is time-varying, then the class of GARCH processes usually employed in the financial literature is misspecified and this class of models should be made more flexible to accommodate these facts.[4]

This paper is organized as follows. The methodology used to evaluate the Hurst's exponent is introduced in Section 2. In Section 3, the data used in this work are presented. In Section 4, the results are discussed. Finally, Section 5 presents some conclusions of this work.

2. Evaluation of Hurst's exponent

In this paper, our measure of long-range dependence is the Hurst exponent provided by the R/S analysis [12,13].[5] More explicitly, let $X(t)$ be the price of a stock

[2]See also [5–7].

[3]This approach has been used to test for time-varying efficiency in the Brazilian stock market, employing variance ratios, in Ref. [9].

[4]Details of GARCH processes may be found in Refs. [10] and [11].

[5]Due to Lo's critique [14] that the R/S statistics is sensitive to the presence of short-range dependence, we also considered as a second methodology a modified version of the R/S analysis considered in Ref. [15] where one applies the R/S analysis to blocks of shuffled data. The effect of random permutations in these small blocks is exactly to destroy any particular structure of autocorrelation within these blocks. However, since the results of this methodology were exactly the same as the previous one, we do not present these results in the paper.

on a time t and $r(t)$ be the logarithmic return denoted by $r(t) = \ln\left(\frac{X(t+1)}{X(t)}\right)$. The R/S statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. So, consider a sample of continuously compounded asset returns $\{r_1, r_2, \ldots, r_\tau\}$ and let $\bar{r}_\tau$ denote the sample mean $\frac{1}{\tau}\sum_\tau r_\tau$, where τ is the time span considered. Then the R/S statistic is given by

$$(R/S)_\tau \equiv \frac{1}{s_\tau}\left[\max_{1\leqslant t\leqslant\tau}\sum_{k=1}^{t}(r_k - \bar{r}_\tau) - \min_{1\leqslant t\leqslant\tau}\sum_{k=1}^{t}(r_k - \bar{r}_\tau)\right], \tag{1}$$

where s_τ is the usual standard deviation estimator

$$s_\tau \equiv \left[\frac{1}{\tau}\sum_t (r_t - \bar{r}_\tau)^2\right]^{\frac{1}{2}}. \tag{2}$$

Hurst found that the rescaled range, R/S, for many records in time is very well described by the following empirical relation:

$$(R/S)_\tau = (\tau/2)^H. \tag{3}$$

3. Data

The sample employed in this study consists of 11 emerging markets and indices for the United States and Japan, which are included for comparison purposes. We have collected daily closing prices for Argentina, Brazil, Chile, India, Indonesia, Malaysia, Mexico, the Philippines, South Korea, Taiwan, Thailand, Japan, and the US. The period comprised in this research stems from January 1991 through January 2004.

We employ Morgan Stanley Capital Index (MSCI) style indices in order to disentangle the effects of autocorrelation and infrequent trading, which is especially acute in emerging markets. These indices focus on the largest capitalization stocks.

A problem that can emerge from market efficiency studies is that one can find evidence of long memory (and short-term autocorrelation) induced by rigid prices since emerging markets have thin trading, i.e., many stock exchanges possess low liquidity, that is reflected in low volatility, which in turn affects market efficiency estimates. Therefore, we use MSCI indices in order to avoid these problems and if evidence suggests that the Hurst exponents are time-varying, this should be attributed to an inherent characteristic of the time series and to more fundamental issues such as differences in market microstructure (differences in types of exchanges, institutions, settlements, and so forth), rather than liquidity problems.

4. Empirical results

We perform the estimation of the Hurst exponent for time windows with 1000 observations each, thousands of times. We use the first 1000 observations, calculate the Hurst exponent, roll the sample one point forward eliminating the first observation and including the next one, calculating the Hurst exponent for the new time window, and repeat this procedure until the end of the series, in a rolling sample approach.[6]

Figs. 1–13 present the empirical results of this paper. In each of them, two plots are presented: the Hurst exponents of absolute returns (our measure of volatility considered in this paper)[7] calculated by means of the R/S analysis for time windows of approximately 4 years (1000 observations) and their respective histograms.

From visual inspection of these figures the first issue that is worth noting is that Hurst exponents seem to be time-varying, as they vary widely over time. This suggests that calculating a Hurst exponent for a time series with a fixed window may be misleading, as most of the literature has done so far.

The histograms suggest that for all series the assumption of normality for the Hurst exponents is strongly rejected. This makes the case that these differences in exponents are not only noise in the data. These results may be confirmed using Table 1, where the Shapiro–Wilk statistic test is applied to the Hurst exponents calculated for the absolute returns. From this table we can see that the normality assumption is strongly rejected for all time series.

These results are in line with the findings of Ref. [8] and suggest that there seems to be moments where long-range predictability increases and others where it decreases. For emerging markets this is a very interesting finding and suggests two lines of research. On one hand, differences in market microstructure conditions for each country may explain the difference obtained in these results for each country. On the other hand, econometric models such as GARCH processes need to become more flexible. The class of fractionally integrated GARCH (FIGARCH) also has to become more flexible as the long-memory parameters seem to be time-varying.[8]

Argentina, Chile, Indonesia, the Philippines, Malaysia, Taiwan, and Thailand seem to have a downward trend in these exponents, which implies a decrease in volatility persistence in this time period. Only India displays an increase in the long-term dependence (upward trend) in volatility.[9]

[6]This procedure was also applied in Ref. [8].

[7]We also calculated the Hurst exponents of squared returns, but since the results are similar to the previous ones, these results are not included in this paper.

[8]Preliminary results in this line may be found in Ref. [16]. Studying and comparing these differences in depth seems an interesting and compelling route for research.

[9]It is important to notice that both absolute and squared returns (which are used as proxies for equity return volatility) share similar characteristics and possess dynamic Hurst exponents. However, there seems to be stronger long-range dependence in absolute returns than in squared returns.

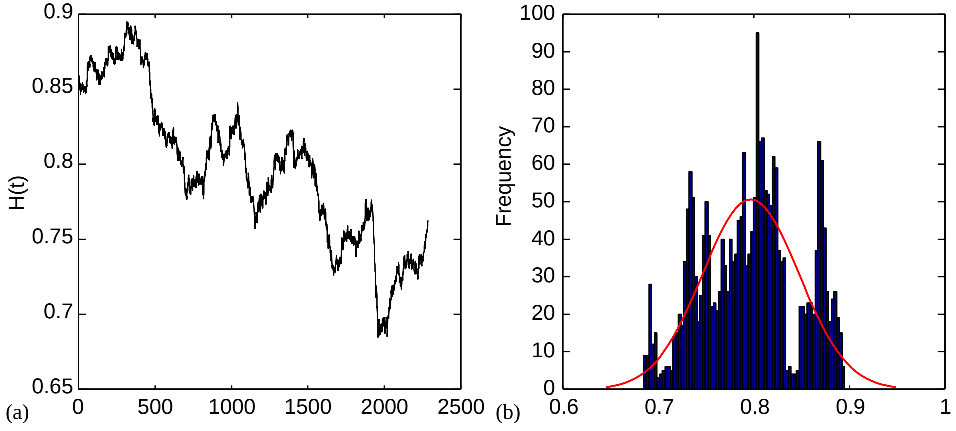

Fig. 1. Plots of time-varying H evaluated for absolute returns (Argentina). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

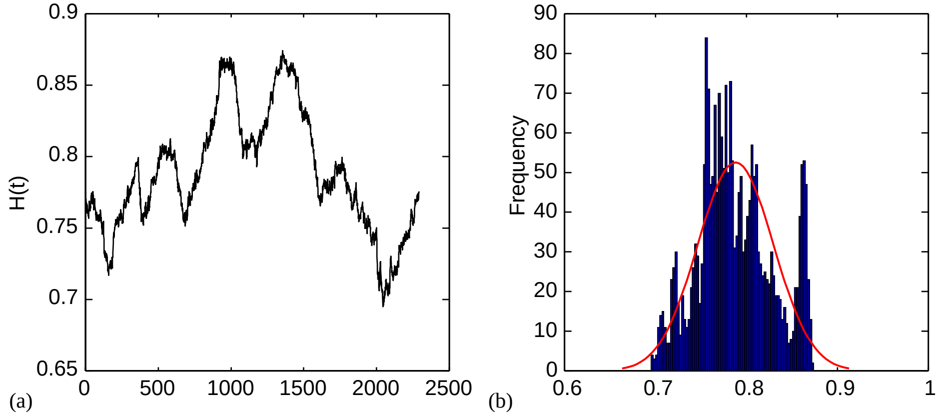

Fig. 2. Plots of time-varying H evaluated for absolute returns (Brazil). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

For developed economies, there is a downward trend in Hurst exponents for Japan, while the opposite holds true for the US.

One could argue that these time-varying effects could be due to short-term autocorrelations as the R/S method is known to be sensitive to this problem (commonly found in emerging market returns and volatility). Therefore, in order to check for the robustness of the results we shuffle squared and absolute returns for

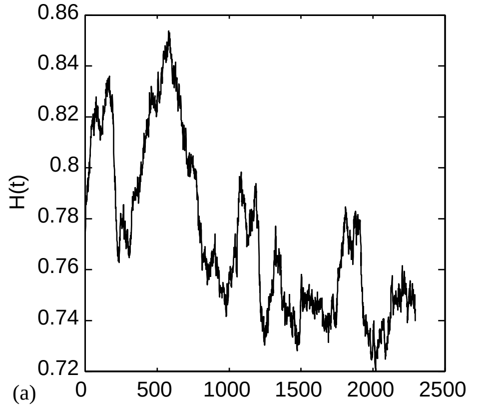

Fig. 3. Plots of time-varying H evaluated for absolute returns (Chile). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

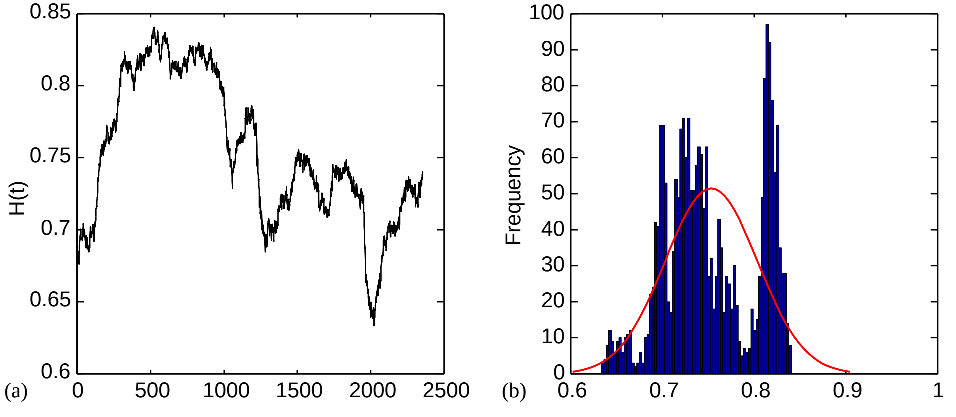

Fig. 4. Plots of time-varying H evaluated for absolute returns (Mexico). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

blocks of a given size across the sample. We have done that for each one of the countries indices analyzed. We employ a block of size 10 and shuffle returns within each block. The results are robust to different block size choices (such as 5 and 15, 20 and 30).

An explanation for these results could be the recent integration of international capital markets, with the release of capital controls, increased export and import (trade balance) to GDP ratios, and issuance of depositary receipts of emerging markets in developed economies. All these may have had an impact in volatility

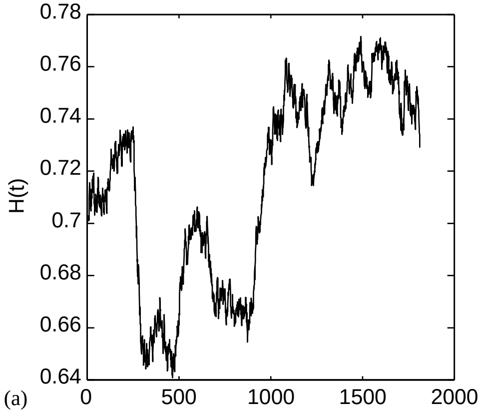

Fig. 5. Plots of time-varying H evaluated for absolute returns (India). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

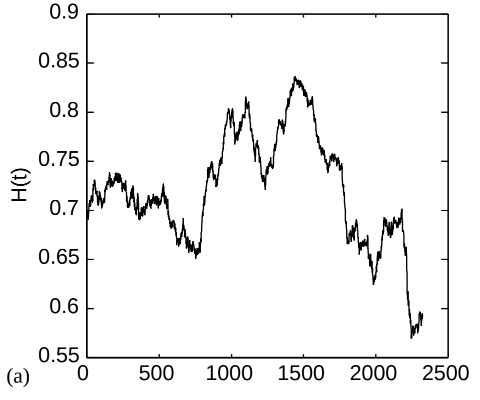

Fig. 6. Plots of time-varying H evaluated for absolute returns (Indonesia). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

dynamics for the equity markets in issue, which could be reflected in a dynamic Hurst exponent.

The recent crisis that equity markets have suffered in recent years may have also played a role on the impact in volatility dynamics. The Asian crisis in 1997 and the Russian crisis in 1998 affected adversely most emerging market economies in our sample, with an increase in the subjacent risk. Therefore, our time-varying measures of long-range dependence are capturing these changes in the dynamics of these

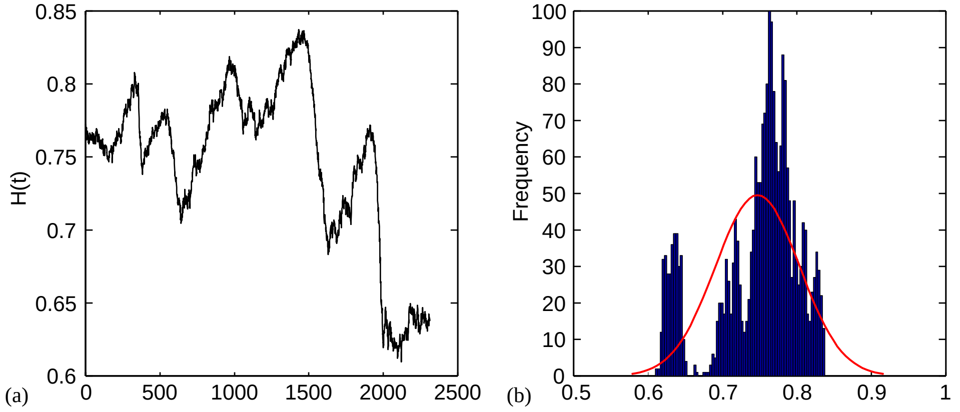

Fig. 7. Plots of time-varying H evaluated for absolute returns (the Philippines). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

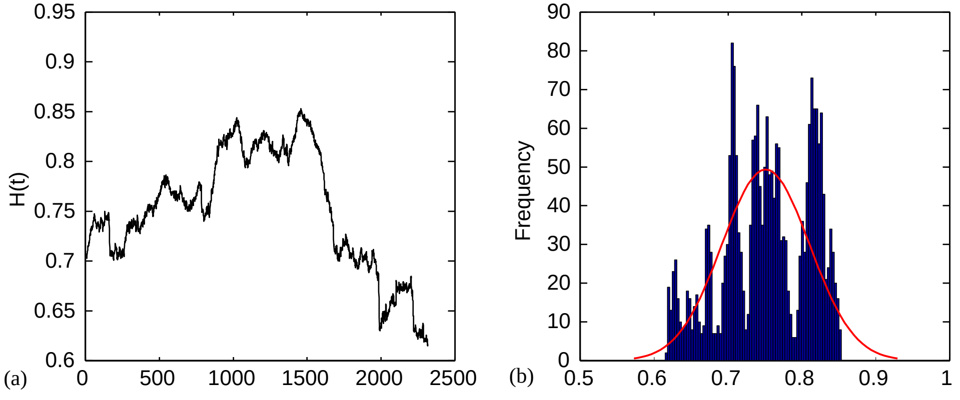

Fig. 8. Plots of time-varying H evaluated for absolute returns (Malaysia). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

volatilities through time due to shocks that have adversely affected these economies, but also fundamentally, due to the changes that have occurred in these economies in the past years, such as increases in openness to international markets, changes in domestic liquidity, and so forth.

A striking feature of these time-varying Hurst exponents is that there seems to be a downward trend among emerging markets in our sample. However, the dynamics seem to be different. Establishing a causal relationship between changes in Hurst

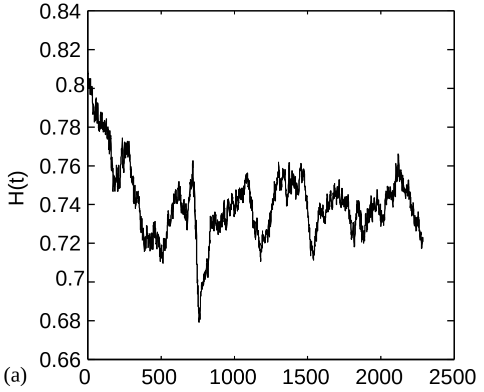

Fig. 9. Plots of time-varying H evaluated for absolute returns (Taiwan). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

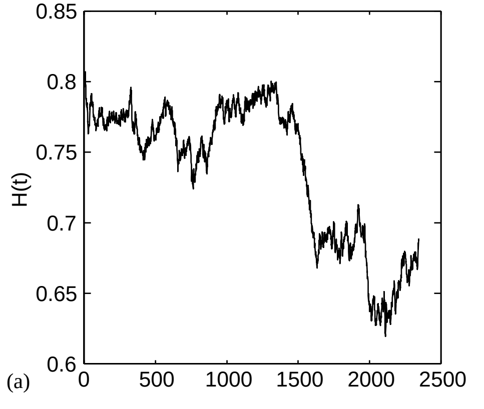

Fig. 10. Plots of time-varying H evaluated for absolute returns (Thailand). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

exponents and market microstructure variables is beyond the scope of this paper but is certainly an important route to be explored in future research.

5. Conclusions

This paper contributes to the literature by showing the importance of studying time-varying Hurst exponents to assess for long-range dependence, instead of relying on single static measures of long-memory dependence.

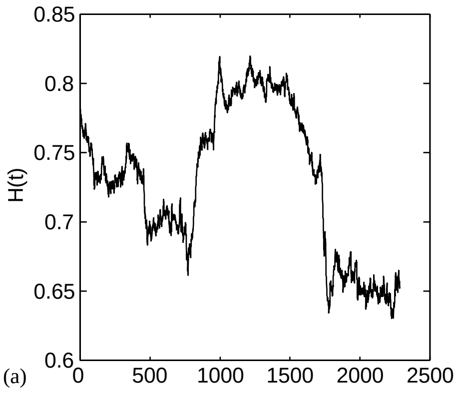

Fig. 11. Plots of time-varying H evaluated for absolute returns (Korea). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

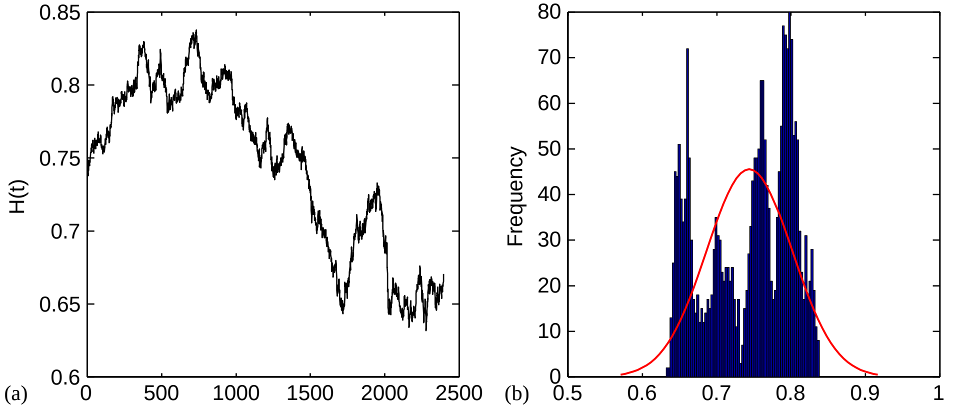

Fig. 12. Plots of time-varying H evaluated for absolute returns (Japan). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

Our results are in line with those found in Ref. [8] and suggest that Hurst exponents vary over time due to changes in the dynamics of the underlying return time series. This is not explained by time-varying short-range dependencies as a shuffling procedure applied to both squared and absolute returns generates qualitatively similar results.

An important route for future research would be explaining the differences in the behavior of these time-varying exponents and characteristics of the time series. These variations could be due to fundamental differences in market microstructure

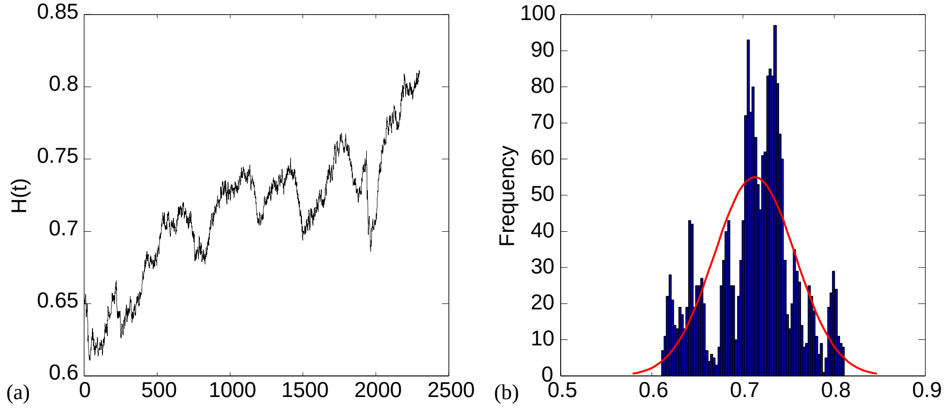

Fig. 13. Plots of time-varying H evaluated for absolute returns (US). In the left-hand side we present (a) Hurst exponents for 1000 observations windows, while in the right-hand side we present (b) a histogram with a normal distribution fit.

Table 1
Shapiro–Wilks normality tests for time-varying Hurst exponents

	Shapiro–Wilk statistic	p-value
Argentina	$W = .97631$	$p<0.0001$
Brazil	$W = .97412$	$p<0.0001$
Chile	$W = .92439$	$p<0.0001$
Mexico	$W = .94500$	$p<0.0001$
India	$W = .92055$	$p<0.0001$
Indonesia	$W = .98225$	$p<0.0001$
Korea	$W = .93623$	$p<0.0001$
Philippines	$W = .90246$	$p<0.0001$
Malaysia	$W = .96311$	$p<0.0001$
Taiwan	$W = .96182$	$p<0.0001$
Thailand	$W = .86061$	$p<0.0001$
Japan	$W = .92002$	$p<0.0001$
US	$W = .96987$	$p<0.0001$

The second and third columns in the table present the Shapiro–Wilks statistics and their respective p-values. The null hypothesis is that these coefficients follow a normal distribution.

variables such as changes in trading patterns, capital flows, trading costs, exchanges, and investor type among others. A deeper analysis of the explanations for these changes should enhance substantially our understanding of equity markets behavior.

The results found in this paper suggest that GARCH models may be misspecified and that more flexible models should be used, allowing for long memory in volatility. Furthermore, it has implications for option pricing and pricing models, which should take into account these stylized facts.

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