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Testing for long range dependence in banking equity indices

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Abstract

This paper presents empirical evidence of long range dependence in returns and volatility for banking indices for 41 different countries. We employ the Rescaled Hurst analysis and develop a formal statistical procedure to test for long range dependence. This procedure allows to rank these countries by relative inefficiency, which can provide guidance for investors and portfolio managers.

1. Introduction

A lot of research has been undertaken in recent years focusing on whether stock returns and volatility possess long range dependence. If there is long range dependence in stock returns then one could improve forecasts in the long term on the dynamics of these time series. Furthermore, the Black–Scholes model is not valid anymore, which is one of the pillars of modern finance. Basically, most of the financial theory relies on the hypothesis that returns do not present long range dependence. This explains why the topic has been a hot subject in the past years. If one can find evidence of long range dependence then we should incorporate this in our pricing models (options pricing) and also in portfolio and risk management (forecasting expected returns and volatility).

Most of the literature has focused on aggregate indices for a variety of countries (see [4,5,7,6,1,2]). However, the literature that focuses on particular sectorial indices is relatively small. Evaluating whether long-range dependence exists in specific sectors of the economy is particularly useful and could help understand the origins of such a phenomena, that seems to be a commonplace in econophysics.

This paper intends to fill this existing gap in the literature by studying long range dependence in banking indices around the world. Our focus on the banking sector relies on the fact that this is certainly one of the most important sectors in the economy. We test for long range dependence for mean returns and volatility for 41 different countries, including developed and emerging economies.

The rest of the paper proceed as follows. In the next section we provide a description of the methodology that is used throughout the paper. Section 3 discusses the data that is used, while Section 4 presents empirical results on long-range dependence measures. Finally, Section 5 presents final remarks and suggestions for further research.

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2. Measures of long-range dependence

In this paper, our measure of long range dependence is the Hurst's exponent provided by the *R*/*S* analysis. We present two different set of results: (1) the *R*/*S* analysis; (2) the *R*/*S* analysis with a shuffling procedure to purge for short term autocorrelation. The shuffling procedure intends to remove any extra long range dependence that may be presented in the data.[1]

The *R*/*S* method (see [10]) due to its simplicity is the most popular way to detect long-range dependence and it is described in which follows. Let $X(t)$ be the price of a stock on a time t and $r(t)$ be the logarithmic return denoted by $r(t) = \ln\left(\frac{X(t+1)}{X(t)}\right)$.

The *R*/*S* statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. So, consider a sample of continuously compounded asset returns $\{r(1), r(2), \ldots, r(\tau)\}$ and let $\bar{r}_\tau$ denote the sample mean $\frac{1}{\tau}\sum_\tau r(\tau)$ where τ is the time span considered. Then the *R*/*S* statistic is given by

$$(R/S)_\tau \equiv \frac{1}{s_\tau}\left[\max_{1\leqslant t\leqslant \tau}\sum_{k=1}^{t}(r(k)-\bar{r}_\tau) - \min_{1\leqslant t\leqslant \tau}\sum_{k=1}^{t}(r(k)-\bar{r}_\tau)\right] \tag{1}$$

where s_τ is the usual standard deviation estimator

$$S_\tau \equiv \left[\frac{1}{\tau}\sum_t (r(t)-\bar{r}_\tau)^2\right]^{\frac{1}{2}} \tag{2}$$

Hurst [10] found that the rescaled range, *R*/*S*, for many records in time is very well described by the following empirical relation:

$$(R/S)_\tau = (\tau/2)^H \tag{3}$$

By means of the *R*/*S* analysis, the Hurst's exponent may be evaluated by plotting the data $(R/S)_\tau$ versus τ in a log–log plot and measuring the slope of the straight line.[2]

Differently from most researchers that present Hurst exponents without an associated standard error, we estimate both the Hurst exponent and it's standard error, which allows for a statistical test on the restriction $H = 0.5$, i.e., we test whether the time series data for which we estimate Hurst exponents have a Hurst exponent statistically significant different from 0.5 (Brownian Motion) and formally test whether these time series are persistent ($H > 0.5$), which implies that past trends are more likely to persist in the future and antipersistent ($H < 0.5$), that suggests that past trends are more likely to change in the future.

We test for ($H = 0.5$) by using a Wald statistic for this restriction. The Wald statistic is given by $W = (H - 0.5)^2/\text{std}^2$, where H is the estimated Hurst exponent and std is its associated standard error. This statistic has a χ^2 distribution with one degree of freedom.

3. Data

Although a lot of work has been done for general indices for equity markets there is a gap in the literature as specific sectorial indices have not yet been subject of study. There are many theoretical reasons on why different sectors in the economy may possess different statistical dynamical properties. Sectors that are pro-cyclical and counter-cyclical should present different underlying dynamics in their time series. Furthermore, in some economies the importance of specific sectors is huge and shocks to these sectors should reflect in changes in the dynamics of other sectors.

In this paper we focus on the banking sector that has not been subject of testing for long range dependence and is one of the most important sectors in the economy due to its impacts on all other sectors and to the recent globalization of financial services.

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[1] Therefore, in (2) and (4), we apply the given method to shuffled data in blocks of pre-determined size, i.e., we pick a random permutation of the data series within each block of predetermined size and apply the *R*/*S* analysis to this shuffled data. The effect of random permutations in these small blocks is to destroy any particular structure of autocorrelation within these blocks, reducing the bias that may arise due to the presence of short-term autocorrelation. Montanari et al. [12] and Willinger et al. [13] discuss the importance of estimating Hurst exponents robust to short-term autocorrelation. See also [11].

[2] For a discussion on testing for long-range dependence, see [9].

We use a data set of 41 banking indices for different countries, including developed economies and emerging markets. The data was provided by DATASTREAM and the period analyzed ranges from January 1995 to December 2003.

Table 1
Hurst exponents for mean, squared and absolute banking equity returns using the *R*/*S* methodology

Country	Without shuffling			With shuffling		
	Returns	Squared	Absolute	Returns	Squared	Absolute
Developed markets	0.526	0.773	0.814	0.525	0.782	0.824
Australia	0.510 0.013	0.677 0.012	0.690 0.012	0.504 0.011	0.690 0.013	0.706 0.014
Austria	0.552 0.011	0.795 0.013	0.824 0.015	0.544 0.011	0.800 0.014	0.833 0.016
Belgium	0.513 0.009	0.801 0.016	0.850 0.018	0.503 0.009	0.814 0.019	0.863 0.020
Canada	0.518 0.009	0.755 0.012	0.792 0.013	0.535 0.008	0.760 0.011	0.801 0.013
Denmark	0.534 0.008	0.757 0.013	0.811 0.015	0.505 0.008	0.773 0.015	0.824 0.014
Finland	0.523 0.008	0.676 0.012	0.754 0.012	0.523 0.008	0.682 0.013	0.760 0.015
France	0.510 0.012	0.838 0.016	0.874 0.017	0.533 0.009	0.840 0.017	0.877 0.018
Germany	0.503 0.012	0.788 0.011	0.824 0.012	0.499 0.011	0.806 0.013	0.838 0.014
Hong Kong	0.567 0.009	0.783 0.019	0.857 0.025	0.566 0.011	0.780 0.019	0.852 0.024
Ireland	0.543 0.008	0.697 0.015	0.733 0.016	0.554 0.009	0.718 0.015	0.762 0.016
Italy	0.569 0.010	0.812 0.012	0.830 0.012	0.551 0.012	0.819 0.013	0.840 0.013
Japan	0.536 0.009	0.784 0.016	0.849 0.021	0.555 0.010	0.771 0.014	0.839 0.018
The Netherlands	0.471 0.013	0.857 0.016	0.879 0.016	0.485 0.011	0.865 0.018	0.890 0.019
Norway	0.546 0.007	0.787 0.015	0.835 0.017	0.526 0.010	0.790 0.016	0.844 0.018
Singapore	0.625 0.012	0.758 0.016	0.842 0.024	0.633 0.015	0.762 0.019	0.842 0.028
Spain	0.511 0.011	0.842 0.011	0.894 0.015	0.509 0.011	0.846 0.013	0.898 0.017
Sweden	0.491 0.008	0.722 0.010	0.730 0.009	0.498 0.008	0.739 0.011	0.747 0.011
Switzerland	0.530 0.009	0.797 0.014	0.836 0.015	0.522 0.009	0.809 0.015	0.846 0.016
United Kingdom	0.508 0.008	0.758 0.011	0.756 0.013	0.507 0.007	0.778 0.012	0.773 0.013
US	0.455 0.016	0.784 0.010	0.825 0.014	0.452 0.016	0.798 0.014	0.838 0.018
Emerging markets	0.578	0.744	0.794	0.581	0.764	0.816
Latin America	0.590	0.735	0.829	0.592	0.756	0.855
Argentina	0.592 0.014	0.738 0.016	0.861 0.020	0.592 0.015	0.748 0.018	0.869 0.023
Brazil	0.566 0.012	0.719 0.015	0.797 0.015	0.595 0.014	0.748 0.017	0.831 0.017
Chile	0.605 0.010	0.694 0.017	0.814 0.020	0.611 0.008	0.727 0.017	0.854 0.020
Mexico	0.567 0.010	0.778 0.015	0.867 0.018	0.565 0.010	0.792 0.016	0.894 0.020
Peru	0.622 0.011	0.746 0.014	0.805 0.020	0.595 0.009	0.765 0.017	0.829 0.021
Asia	0.601	0.753	0.804	0.613	0.773	0.830
India	0.535 0.012	0.731 0.009	0.753 0.010	0.541 0.009	0.745 0.010	0.769 0.012
Indonesia	0.595 0.013	0.785 0.032	0.827 0.037	0.607 0.015	0.806 0.031	0.855 0.035
Korea	0.568 0.007	0.806 0.024	0.838 0.031	0.587 0.007	0.832 0.025	0.866 0.034
Malaysia	0.648 0.014	0.771 0.016	0.885 0.028	0.670 0.012	0.790 0.018	0.912 0.030
The Philippines	0.684 0.012	0.723 0.018	0.789 0.026	0.697 0.012	0.750 0.020	0.820 0.027
Taiwan	0.571 0.010	0.694 0.007	0.724 0.009	0.573 0.008	0.723 0.008	0.757 0.011
Thailand	0.599 0.010	0.832 0.033	0.867 0.037	0.617 0.009	0.847 0.034	0.886 0.038
Sri Lanka	0.609 0.011	0.678 0.010	0.751 0.011	0.613 0.010	0.694 0.011	0.778 0.013
Europe	0.569	0.756	0.797	0.577	0.782	0.827
Czech Republic	0.621 0.010	0.787 0.016	0.825 0.020	0.635 0.012	0.809 0.018	0.850 0.023
Greece	0.563 0.012	0.794 0.022	0.845 0.025	0.572 0.012	0.823 0.023	0.874 0.027
Hungary	0.546 0.009	0.704 0.007	0.749 0.009	0.543 0.008	0.731 0.008	0.787 0.011
Poland	0.556 0.015	0.799 0.021	0.821 0.021	0.562 0.014	0.826 0.021	0.856 0.019
Portugal	0.602 0.007	0.771 0.019	0.822 0.023	0.604 0.008	0.799 0.019	0.857 0.022
Turkey	0.528 0.012	0.682 0.009	0.720 0.009	0.548 0.010	0.706 0.009	0.736 0.009
Middle East	0.55	0.73	0.75	0.54	0.74	0.75
Israel	0.558 0.010	0.702 0.010	0.754 0.010	0.545 0.011	0.709 0.010	0.754 0.010
Pakistan	0.543 0.010	0.761 0.012	0.741 0.010	0.542 0.008	0.780 0.014	0.751 0.011

By studying such a wide variety of countries it is possible to ascertain with more precision whether the banking sector, on average, seems to possess long range dependence and compare different countries in terms of long term predictability. This has been done for emerging markets general indices in [5] and [7].

Table 2
Wald statistics for the restriction $H = 0.5$ for mean, squared and absolute banking equity returns

Country	Returns	Squared	Absolute	Returns	Squared	Absolute
Developed markets	19.258	444.242	448.088	14.562	394.156	404.294
Australia	0.64 0.42	224.67 0.00	257.76 0.00	0.15 0.70	207.98 0.00	214.59 0.00
Austria	20.94 0.00	512.98 0.00	491.01 0.00	15.73 0.00	444.10 0.00	417.18 0.00
Belgium	2.10 0.15	338.47 0.00	384.96 0.00	0.11 0.74	281.49 0.00	320.49 0.00
Canada	4.27 0.04	482.84 0.00	488.87 0.00	18.57 0.00	534.29 0.00	513.79 0.00
Denmark	18.87 0.00	393.98 0.00	415.32 0.00	0.48 0.49	343.71 0.00	532.21 0.00
Finland	9.60 0.00	203.57 0.00	416.17 0.00	9.60 0.00	186.69 0.00	307.32 0.00
France	0.75 0.39	420.92 0.00	480.98 0.00	12.30 0.00	381.95 0.00	445.80 0.00
Germany	0.06 0.80	658.85 0.00	722.37 0.00	0.01 0.92	584.50 0.00	581.47 0.00
Hong Kong	54.21 0.00	228.67 0.00	201.52 0.00	34.18 0.00	220.64 0.00	213.78 0.00
Ireland	27.97 0.00	180.39 0.00	216.87 0.00	39.66 0.00	222.84 0.00	275.20 0.00
Italy	52.81 0.00	646.00 0.00	716.40 0.00	18.26 0.00	602.65 0.00	665.56 0.00
Japan	16.21 0.00	333.49 0.00	281.57 0.00	33.61 0.00	402.88 0.00	341.88 0.00
The Netherlands	4.65 0.03	471.40 0.00	530.08 0.00	1.71 0.19	408.61 0.00	419.31 0.00
Norway	39.94 0.00	366.38 0.00	377.34 0.00	7.56 0.01	344.11 0.00	366.41 0.00
Singapore	110.59 0.00	252.33 0.00	201.50 0.00	81.17 0.00	192.28 0.00	152.21 0.00
Spain	0.99 0.32	888.23 0.00	679.19 0.00	0.71 0.40	664.19 0.00	553.93 0.00
Sweden	1.38 0.24	456.21 0.00	631.37 0.00	0.06 0.81	486.90 0.00	530.18 0.00
Switzerland	10.24 0.00	476.03 0.00	525.30 0.00	6.63 0.01	442.25 0.00	475.33 0.00
United Kingdom	0.97 0.33	590.61 0.00	407.56 0.00	1.18 0.28	501.50 0.00	416.86 0.00
US	7.97 0.00	758.81 0.00	535.61 0.00	9.56 0.00	429.57 0.00	342.37 0.00
Emerging markets	65.064	335.581	395.873	77.669	339.316	392.523
Latin America	72.250	241.967	318.376	86.877	239.608	316.158
Argentina	45.04 0.00	223.01 0.00	310.93 0.00	35.71 0.00	199.51 0.00	260.02 0.00
Brazil	29.90 0.00	205.61 0.00	396.09 0.00	49.79 0.00	204.93 0.00	383.44 0.00
Chile	111.97 0.00	132.20 0.00	244.20 0.00	191.99 0.00	184.00 0.00	323.75 0.00
Mexico	48.43 0.00	349.74 0.00	396.46 0.00	41.38 0.00	353.65 0.00	373.87 0.00
Peru	125.91 0.00	299.28 0.00	244.20 0.00	115.52 0.00	255.96 0.00	239.71 0.00
Asia	93.642	325.353	283.416	131.738	316.096	274.806
India	8.13 0.00	675.20 0.00	591.85 0.00	20.48 0.00	620.83 0.00	484.00 0.00
Indonesia	56.69 0.00	79.90 0.00	77.98 0.00	51.82 0.00	98.52 0.00	101.69 0.00
Korea	104.17 0.00	168.94 0.00	116.66 0.00	168.25 0.00	171.33 0.00	116.77 0.00
Malaysia	105.54 0.00	288.36 0.00	184.78 0.00	188.61 0.00	267.25 0.00	183.92 0.00
The Philippines	237.82 0.00	151.51 0.00	123.41 0.00	269.60 0.00	159.75 0.00	142.05 0.00
Taiwan	51.34 0.00	784.95 0.00	569.09 0.00	79.03 0.00	768.70 0.00	590.09 0.00
Thailand	90.62 0.00	101.88 0.00	98.43 0.00	157.03 0.00	106.21 0.00	104.33 0.00
Sri Lanka	94.82 0.00	352.09 0.00	505.12 0.00	119.08 0.00	336.17 0.00	475.60 0.00
Europe	68.463	344.044	370.280	70.238	378.608	393.269
Czech Republic	136.02 0.00	329.08 0.00	269.43 0.00	134.20 0.00	303.96 0.00	235.60 0.00
Greece	30.05 0.00	181.51 0.00	189.84 0.00	36.60 0.00	198.23 0.00	193.14 0.00
Hungary	26.65 0.00	768.85 0.00	774.22 0.00	26.22 0.00	813.12 0.00	686.95 0.00
Poland	13.45 0.00	194.46 0.00	244.78 0.00	19.38 0.00	237.98 0.00	342.55 0.00
Portugal	199.41 0.00	207.25 0.00	197.87 0.00	181.34 0.00	249.91 0.00	258.37 0.00
Turkey	5.19 0.02	383.11 0.00	545.54 0.00	23.68 0.00	468.45 0.00	642.95 0.00
Middle East	25.90	430.96	611.42	21.83	422.95	585.87
Israel	33.95 0.00	393.24 0.00	602.61 0.00	15.36 0.00	426.57 0.00	668.27 0.00
Pakistan	17.85 0.00	468.68 0.00	620.23 0.00	28.29 0.00	419.34 0.00	503.47 0.00

4. Empirical results

Table 1 presents the Hurst exponents for mean returns and volatility.[3] The first set of results (in the left hand side) are provided for unshuffled returns, and therefore no correction for short-term autocorrelation was done. In the right hand side we present results for shuffled returns.[4] We present in each case both the estimated Hurst exponent and its associated standard error.

In Table 1 we present average Hurst exponents for developed markets and emerging markets, which are 0.526 and 0.578 using unshuffled returns and 0.525 and 0.581, respectively, suggesting that on average there is more long term predictability in emerging markets than in developed economies. However, this results change when we evaluate average Hurst exponents for volatility (independent on how we construct our measure of volatility). Hurst exponents are on average 0.814 and 0.824 for developed economies (unshuffled and shuffled returns), respectively, and 0.794 and 0.816 for emerging markets.

These results suggest that there is strong long term persistence in volatility for banking sector indices around the world for volatility.

Table 2 presents Wald statistics for testing the restriction $H = 0.5$ on the estimated Hurst exponent. For unshuffled returns we cannot reject the null hypothesis of no long-range dependence for Australia, Belgium, France, Germany, Spain, Sweden and the UK, while for shuffled returns we cannot reject the null hypothesis for Australia, Belgium, Denmark, Germany, the Netherlands, Spain, Sweden and the UK. Furthermore, we reject the null of no long-range dependence in banking sector returns for all emerging markets. These results suggest that emerging markets possess a stronger degree of predictability than developed markets. However, empirical results for testing for long-range dependence in volatility suggest that irrespective of market type there is a strong degree of long-range dependence in volatility.

5. Final remarks

This paper tests for long-range dependence in banking sector indices for 41 different countries around the world. Empirical results suggest that there is a stronger degree of long-range dependence in equity returns for emerging markets than for developed economies. Furthermore, long-range dependence in volatility seems to be stronger in developed economies than in emerging markets, on average.

These results suggest that pricing option models should incorporate long-range dependence in their formulation, which should enhance both portfolio and risk management. Additionally, GARCH models and their variants should be replaced by conditional variance models that incorporate long-term memory parameters.[5]

Additional research is needed in order to understand why different countries possess different degrees of predictability. This could be due to differences in market microstructure or institutional arrangements. Testing for what drives these results is a topic of somewhat great importance. Finally, incorporating transition economies would increase or knowledge of the origins of long-range dependence and is certainly a topic that should require more attention.

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[3] Two proxies were used to construct the volatility series. First, we use absolute returns. Secondly, squared returns.
[4] Following [7,6] we use a block size of 20 observations. Nonetheless, results are robust to using different block sizes such as 10 and 30.
[5] See [8,3].

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