Abstract

This paper employs a “rolling sample” approach to estimate Hurst exponents for emerging markets squared and absolute returns. The findings suggest that these markets possess strong long-range dependence in volatility. Empirical results suggest that Asian equity markets are more efficient than those of Latin America and that the US is the most efficient country.

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1. Introduction

The presence of long memory dependence in asset returns has been subject of a long and extensive research. In a recent paper, Cajueiro and Tabak [2] have shown that long-range dependence for equity returns is time-varying and therefore the dynamics of these Hurst exponents should be explored. Furthermore, Cajueiro and Tabak [3] presented a rank for efficiency built by analyzing median Hurst exponents for different countries. The authors have suggested that Asian equity returns are more inefficient than Latin American equity markets (with the exception of Chile) as these countries possess greater median Hurst exponents.

In this paper we employ a similar approach but focus on testing for long-range dependence for volatility of equity returns. If volatility possesses long memory then most commonly used volatility models such as ARCH and GARCH models, introduced respectively by Engle [5] and Bollerslev [1], are misspecified. In fact, there are other motivations to approach the long-range dependence phenomena in volatility: (a) volatility is directly related to the amount of information arriving in the financial market at a given time and (b) volatility is a very important measure of risk. Furthermore, predictability in volatility means that one can improve forecasts on underlying assets in option markets, contradicting evidence of efficiency.

The paper is organized as follows: the methodology used to evaluate the Hurst’s exponent is introduced in Section 2; in Section 3, we present the data used in this work; in Section 4, the empirical results are discussed; finally, Section 5 presents some conclusions of this work.

2. Evaluation of the Hurst’s exponent

In this paper, the evaluation of the Hurst’s exponent is carried out by two methodologies: the first one is the usual and most popular methodology introduced by Hurst [7] in which the $R/S$ analysis is applied to the log return time series to evaluate the Hurst’s exponent. More explicitly, let $X(t)$ be the price of a stock on a time $t$ and $r(t)$ be the logarithmic
return denoted by $r(t) = \ln \left( \frac{X(t+1)}{X(t)} \right)$. The $R/S$ statistic is the range of partial sums of deviations of times series from its mean, rescaled by its standard deviation. So, consider a sample of continuously compounded asset returns \{\(r_1, r_2, \ldots, r_t\)\} and let $\bar{r}_t$ denote the sample mean \(\frac{1}{t} \sum r_t\) where $\tau$ is the time span considered. Then the $R/S$ statistic is given by

\[
(R/S)_\tau = \frac{1}{s_\tau} \left[ \max_{1 \leq t \leq \tau} \sum_{k=1}^{t} (r_k - \bar{r}_t) - \min_{1 \leq t \leq \tau} \sum_{k=1}^{t} (r_k - \bar{r}_t) \right]
\]

where $s_\tau$ is the usual standard deviation estimator

\[
s_\tau \equiv \left[ \frac{1}{\tau} \sum (r_t - \bar{r}_t)^2 \right]^{\frac{1}{2}}
\]

Hurst found that the rescaled range, $R/S$, for many records in time is very well described by the following empirical relation:

\[
(R/S)_\tau = (\tau/2)^{H}
\]

The second methodology is a modified version of the $R/S$ analysis considered in [6], where one applies the $R/S$ analysis to blocks of shuffled data, i.e., one picks a random permutation of the data series within blocks of predetermined size (in general, small size blocks) and applies the $R/S$ analysis to this shuffled data. \(^1\) In this work, we use blocks of size 10. \(^2\)

3. Data

The sample employed in this study consists of eleven emerging markets and indices for the United States and Japan, which are included for comparison purposes. We have collected daily closing prices for Argentina, Brazil, Chile, India, Indonesia, Malaysia, Mexico, the Philippines, South Korea, Taiwan, Thailand, Japan, and the US. The sample period stems from January 1991 through January 2004.

4. Empirical results

We perform the estimation of the Hurst exponent for time windows with 1000 observations each, thousands of times. We use the first 1000 observations, calculate the Hurst exponent, roll the sample one point forward eliminating the first observation and including the next one, calculating the Hurst exponent for the new time window, and repeat this procedure until the end of the series, in a rolling sample approach. \(^3\)

Table 1 presents the descriptive statistics for Hurst exponents for squared equity returns (proxy for volatility). As we can see these Hurst exponents are high ranging from 0.68 for Indonesia to 0.75 (Argentina and Brazil). Another commonly used proxy for volatility are absolute returns. If we employ this proxy, we can see from Table 2 that Hurst exponents are higher (ranging from 0.717 to 0.8), but that qualitative results remain similar. These results are quite different from the ones found for equity returns in [3]. In [3], analyzing equity returns, the authors suggested that Asian equity returns are more inefficient than Latin American equity markets (with the exception of Chile) as these countries possess greater median Hurst exponents. So, one important issue which is not approached here is to explain the sources of these differences. Which are the properties of these markets which affect differently predictability in volatility and equity returns? We believe that differences in microstructure market conditions may be used to explain these findings. \(^4\)

The main problem with the use of the $R/S$ analysis is that it is sensitive to short-term autocorrelation. It is a widely known stylized fact that volatility possesses short-term autocorrelation for financial time series. Therefore, we employ

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\(^1\) This is justified due to the Lo’s critique [8] that the $R/S$ statistics is sensitive to the presence of short range dependence and the effect of random permutations in these small blocks is exactly to destroy any particular structure of autocorrelation within these blocks.

\(^2\) Our results are robust to different block size choices such as 5, 15, 20 and 30.

\(^3\) This procedure was also applied in [2,3].

\(^4\) Preliminary results in this line may be found in [4].
the R/S analysis for shuffled squared and absolute returns. Results are presented in Tables 3 and 4. As we can see qualitatively, results remain the same.

In Tables 1–4 we also present the Jarque-Bera test for normality (we reject normality assumption for all time series, which suggests that these Hurst exponents are indeed time-varying) and the number of Hurst exponents calculated for each index (observations). The number of observations is different for each country due to differences in holidays among these countries.

In Table 5 we rank these indices using medians for the calculated Hurst exponents. It is worth noting that after taking into account short-term autocorrelation the US are the most efficient market, but that the Hurst exponent is high (0.678–0.708, with squared and absolute returns, respectively). Japan is located in the middle, irrespective of the procedure to calculate volatility and to controlling for short-term autocorrelation. The striking feature is that Mexico differs from other Latin American countries as it is more efficient than Chile, Argentina and Brazil, which are the most inefficient ones.

In order to test whether these rankings are meaningful we employ nonparametric tests for equality of medians (Table 6). All five tests employed lead to the same conclusion, we can reject the null of no differences in these medians at the 1% significance level.

### Table 1
Descriptive statistics for Hurst exponents for squared returns

<table>
<thead>
<tr>
<th></th>
<th>ARG</th>
<th>BRA</th>
<th>CHI</th>
<th>MX</th>
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<th>JP</th>
<th>US</th>
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</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.748</td>
<td>0.751</td>
<td>0.750</td>
<td>0.711</td>
<td>0.700</td>
<td>0.673</td>
<td>0.695</td>
<td>0.717</td>
<td>0.715</td>
<td>0.702</td>
<td>0.701</td>
<td>0.715</td>
<td>0.689</td>
</tr>
<tr>
<td>Median</td>
<td>0.75</td>
<td>0.75</td>
<td>0.74</td>
<td>0.701</td>
<td>0.71</td>
<td>0.68</td>
<td>0.713</td>
<td>0.735</td>
<td>0.715</td>
<td>0.723</td>
<td>0.704</td>
<td>0.721</td>
<td>0.689</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.86</td>
<td>0.81</td>
<td>0.82</td>
<td>0.795</td>
<td>0.76</td>
<td>0.757</td>
<td>0.776</td>
<td>0.792</td>
<td>0.803</td>
<td>0.772</td>
<td>0.784</td>
<td>0.792</td>
<td>0.782</td>
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<tr>
<td>Minimum</td>
<td>0.62</td>
<td>0.68</td>
<td>0.68</td>
<td>0.622</td>
<td>0.63</td>
<td>0.535</td>
<td>0.586</td>
<td>0.588</td>
<td>0.65</td>
<td>0.585</td>
<td>0.61</td>
<td>0.626</td>
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<td>SD</td>
<td>0.062</td>
<td>0.028</td>
<td>0.031</td>
<td>0.046</td>
<td>0.038</td>
<td>0.045</td>
<td>0.050</td>
<td>0.051</td>
<td>0.031</td>
<td>0.046</td>
<td>0.046</td>
<td>0.045</td>
<td>0.037</td>
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<tr>
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<td>0.078</td>
<td>-0.161</td>
<td>0.394</td>
<td>0.209</td>
<td>-0.287</td>
<td>-0.643</td>
<td>-0.526</td>
<td>-0.989</td>
<td>0.547</td>
<td>-0.611</td>
<td>-0.128</td>
<td>-0.187</td>
<td>0.377</td>
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<td>0.0002</td>
<td>0.0000</td>
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<td>0.0000</td>
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<tr>
<td>Observations</td>
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<td>2292</td>
<td>2295</td>
<td>2353</td>
<td>1812</td>
<td>2325</td>
<td>2315</td>
<td>2312</td>
<td>2287</td>
<td>2345</td>
<td>2284</td>
<td>2395</td>
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### Table 2
Descriptive statistics for Hurst exponents for absolute returns

<table>
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<tr>
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<th>TH</th>
<th>KO</th>
<th>JP</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.797</td>
<td>0.788</td>
<td>0.773</td>
<td>0.753</td>
<td>0.751</td>
<td>0.725</td>
<td>0.751</td>
<td>0.747</td>
<td>0.741</td>
<td>0.738</td>
<td>0.729</td>
<td>0.744</td>
<td>0.713</td>
</tr>
<tr>
<td>Median</td>
<td>0.8</td>
<td>0.78</td>
<td>0.77</td>
<td>0.744</td>
<td>0.72</td>
<td>0.717</td>
<td>0.753</td>
<td>0.761</td>
<td>0.76</td>
<td>0.763</td>
<td>0.74</td>
<td>0.734</td>
<td>0.758</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.89</td>
<td>0.87</td>
<td>0.85</td>
<td>0.841</td>
<td>0.77</td>
<td>0.837</td>
<td>0.853</td>
<td>0.837</td>
<td>0.81</td>
<td>0.807</td>
<td>0.82</td>
<td>0.838</td>
<td>0.811</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.68</td>
<td>0.7</td>
<td>0.72</td>
<td>0.633</td>
<td>0.64</td>
<td>0.57</td>
<td>0.615</td>
<td>0.619</td>
<td>0.619</td>
<td>0.619</td>
<td>0.619</td>
<td>0.63</td>
<td>0.632</td>
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<tr>
<td>SD</td>
<td>0.051</td>
<td>0.042</td>
<td>0.033</td>
<td>0.050</td>
<td>0.037</td>
<td>0.057</td>
<td>0.059</td>
<td>0.056</td>
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<tr>
<td>Kurtosis</td>
<td>-0.029</td>
<td>0.214</td>
<td>0.641</td>
<td>0.025</td>
<td>-0.347</td>
<td>-0.155</td>
<td>-0.295</td>
<td>-0.886</td>
<td>0.564</td>
<td>-0.740</td>
<td>-0.151</td>
<td>-0.395</td>
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<tr>
<td>Jarque-Bera</td>
<td>2.240</td>
<td>2.401</td>
<td>2.298</td>
<td>1.989</td>
<td>1.763</td>
<td>2.823</td>
<td>2.220</td>
<td>2.961</td>
<td>4.558</td>
<td>2.105</td>
<td>1.752</td>
<td>1.816</td>
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</tr>
<tr>
<td>Probability</td>
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</tr>
</tbody>
</table>
This paper contributes to the literature by showing the importance of studying time-varying Hurst exponents to assess for long-range dependence, instead of relying on single static measures of long memory dependence. Furthermore, we rank efficiency for equity squared and absolute returns (volatility) for emerging markets and compared to developed economies (US and Japan). The main conclusion is that Asian countries are more efficient than those of Latin America (with the exception of Mexico). Furthermore, there is strong evidence of long memory in equity volatility.

These facts suggest that one should employ this information in options markets to forecast equity returns for equity indices. Moreover, option models that incorporate long memory characteristics in volatility are particularly welcome.

References